



KATHOLIEKE UNIVERSITEIT  
**LEUVEN**

XIX CNIM 15-16/11 Castellón



# Structural Dynamic Behaviour of Tyres

**Paul Sas**

Noise & Vibration Engineering Research Group

KU.Leuven,

Dept. Mechanical Engineering, Div. PMA

# Road traffic noise

- Vehicle noise:

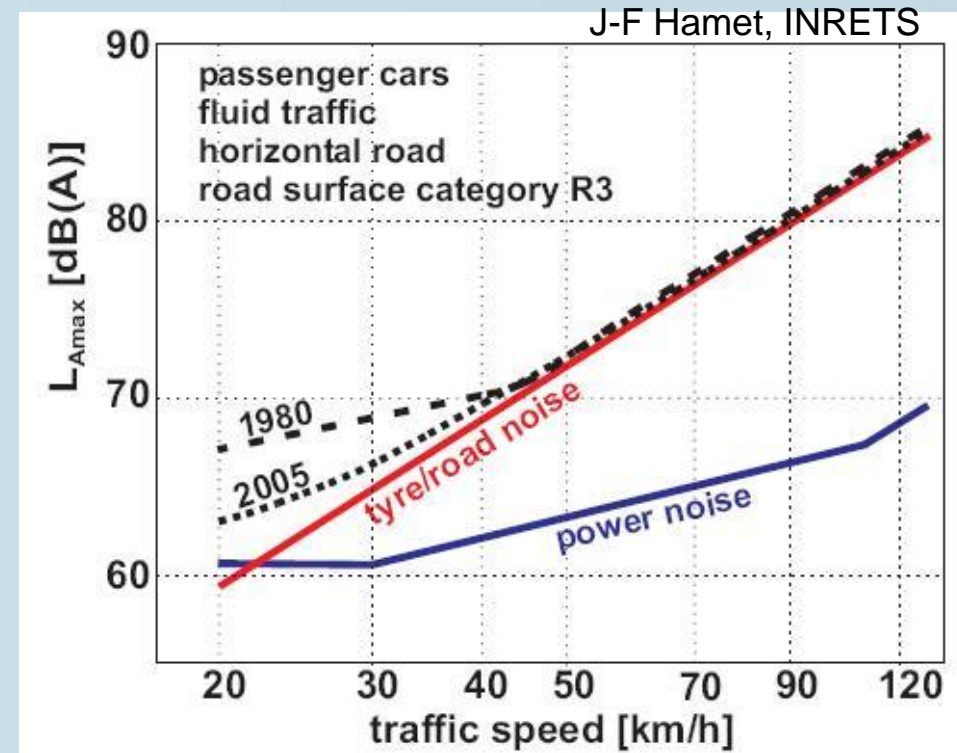
- *drive train noise*

- (engine, intake, exhaust, transmission)

- *aerodynamic noise*

- *tyre/road noise*

**Tyre/road noise dominates  
at constant speeds above  
15-25 km/h**



# Tyre/road noise legislation

- **Directive 2001/43/EC**

(80 km/h coast-by on ISO10844 road surface)

tyre class	CURRENT		PROPOSED	
	tyre section width [mm]	limit value [dB(A)]	tyre section width [mm]	proposed reduction [dB(A)]
C1a	≤145	72	≤185	2.5-4.5
C1b	>145 ≤165	73	>185 ≤215	4.5
C1c	>165 ≤185	74	>215 ≤245	5.5
C1d	>185 ≤215	75	>245 ≤275	4.5
C1e	>215	76	>275	2.5

**70**

**71**

**71**

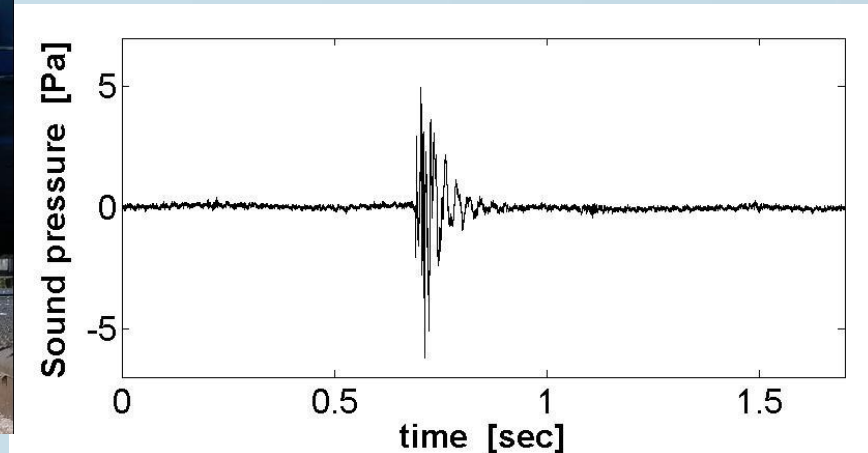
**72**

**74**

- 1) expected to come into force in 2012
- 2) more realistic reference road surface

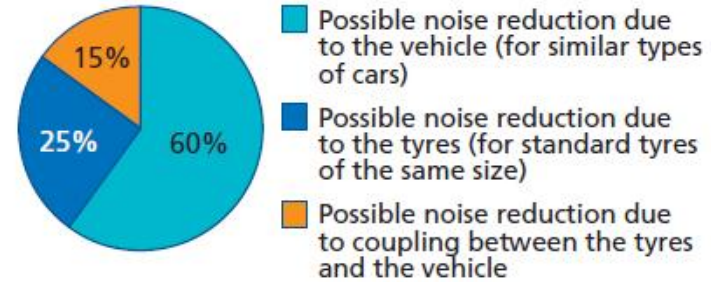
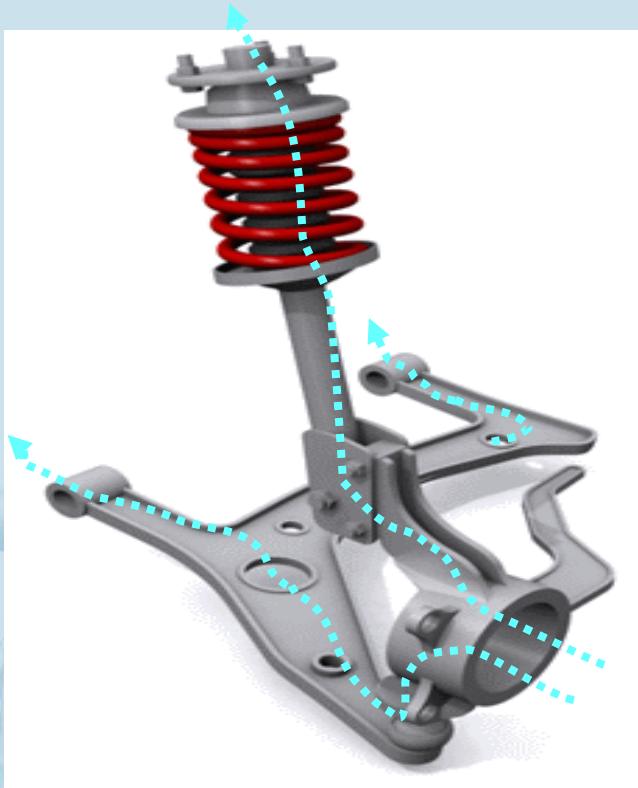
# Crossing a road surface discontinuity

- Tyre/road noise **EXTERIOR** to the vehicle:
  - significant increase of instantaneous noise emission level
  - transient noise is perceived as highly annoying
  - demand for more quiet tyres and road surfaces in urban areas



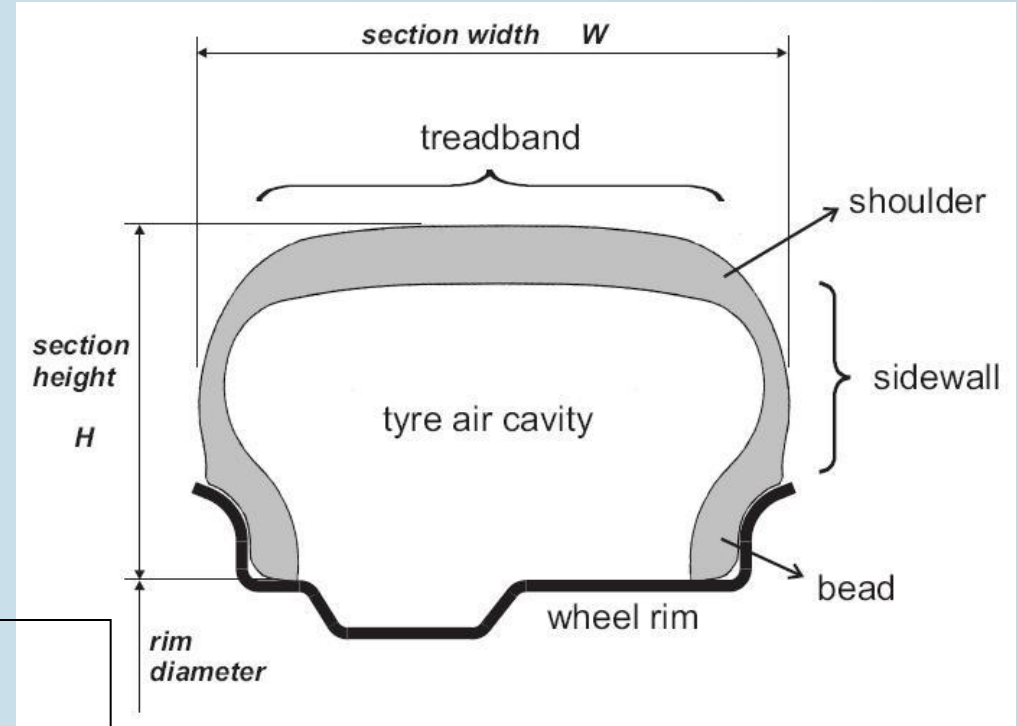
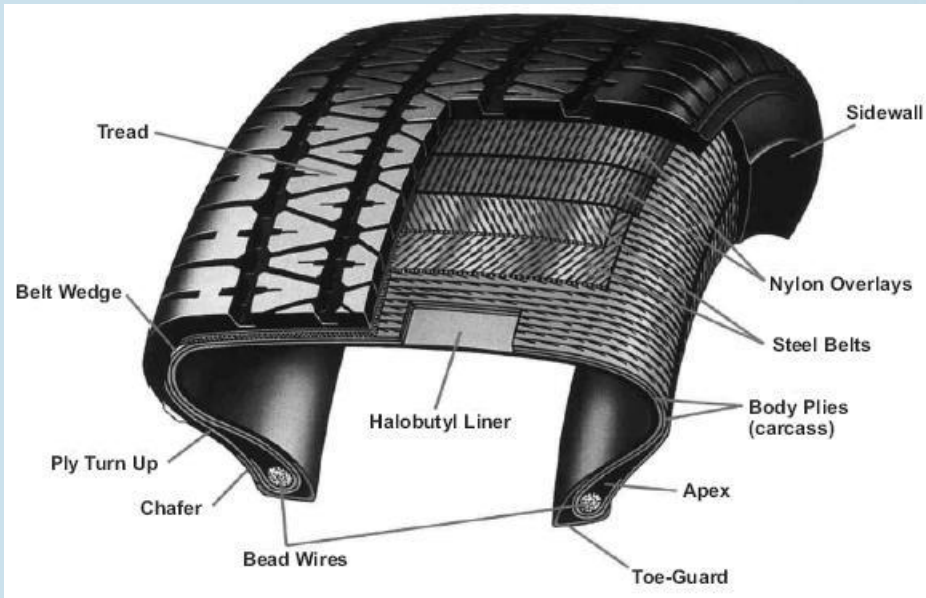
# Relevance of tire dynamics

- Interior vehicle noise (NVH):





# The pneumatic tyre

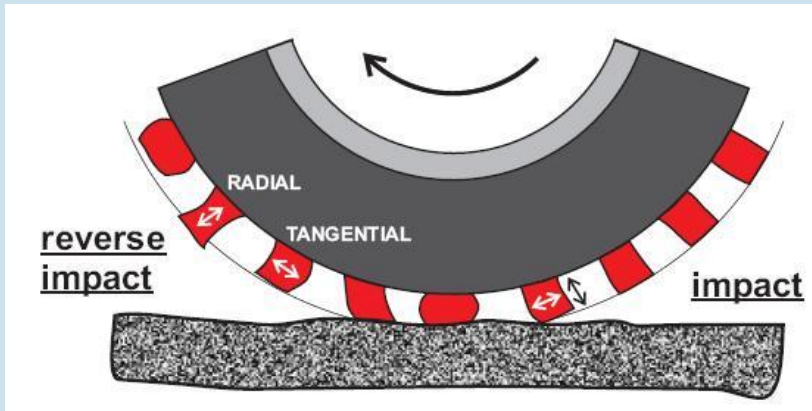


Typical passenger car tyre contains:

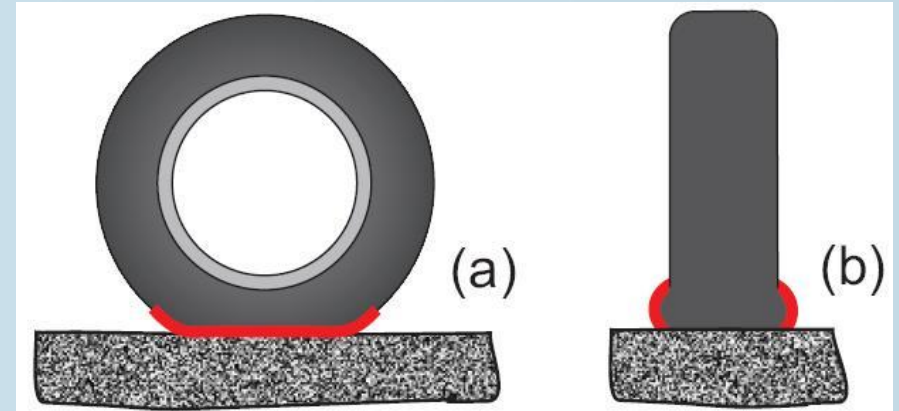
- 13 different types of rubber compounds
- 8 types of fillers (carbon black, silica)
- reinforcement: steel cords, polyester, nylon, rayon
- 40 different kinds of chemicals

approximately 500  
different  
specifications

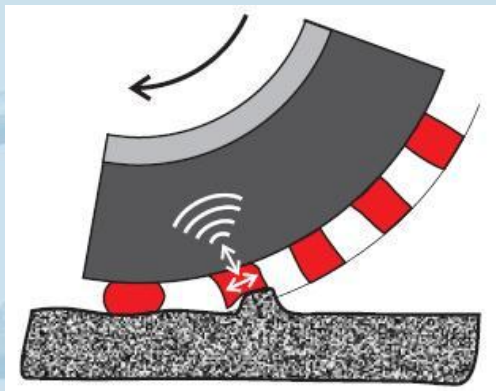
# Tyre/road noise – VIBRATIONAL phenomena



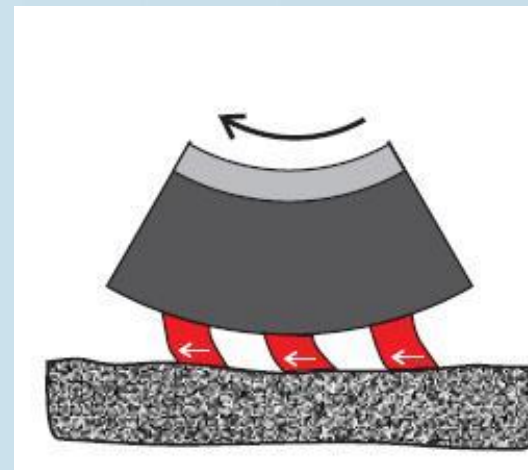
Tread element impact



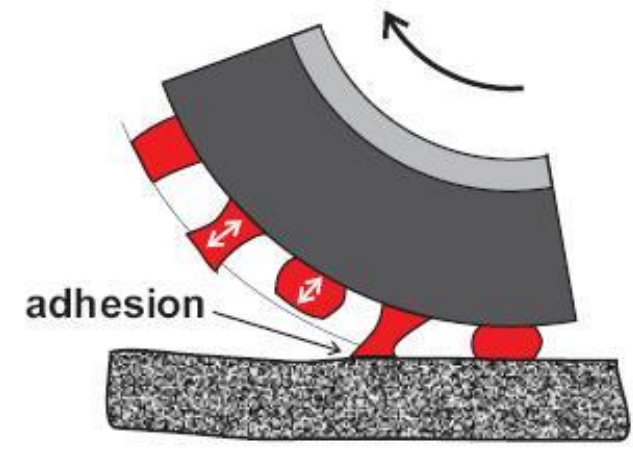
Running deflections



Road texture impact

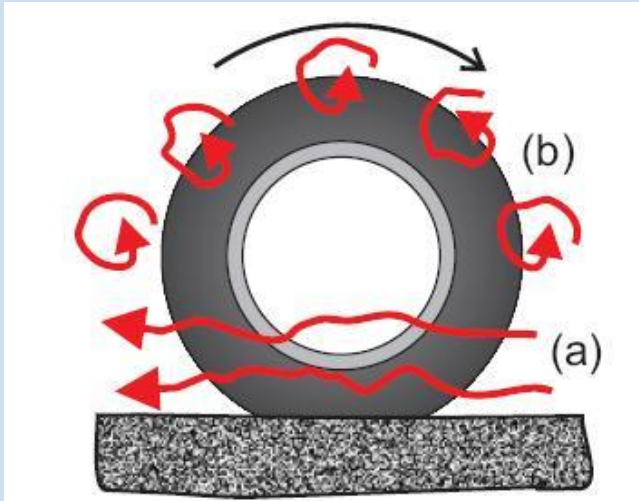


Stick-slip adhesion

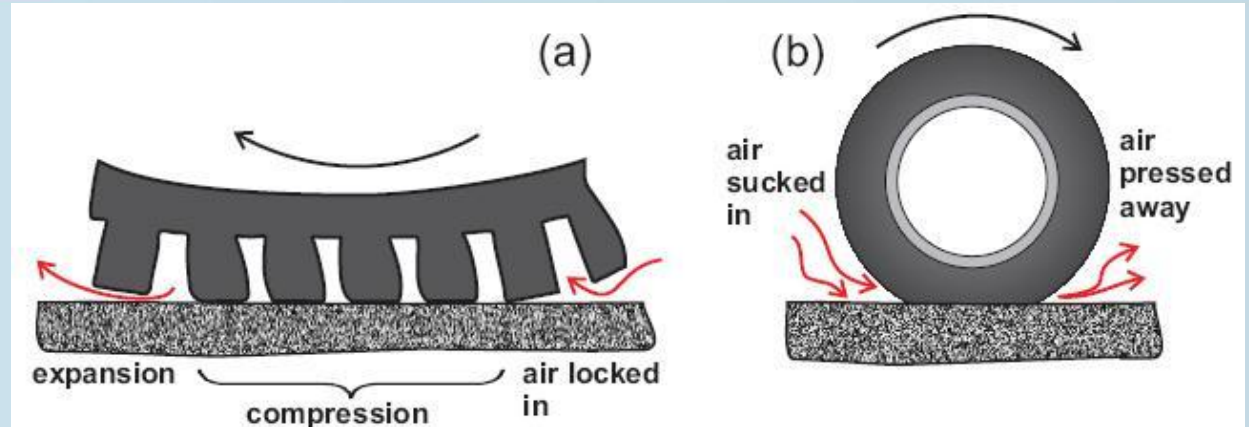


Stick-snap adhesion

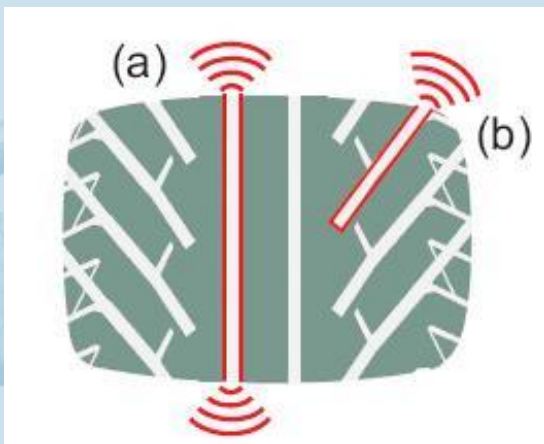
# Tyre/road noise – AERODYNAMICAL phenomena



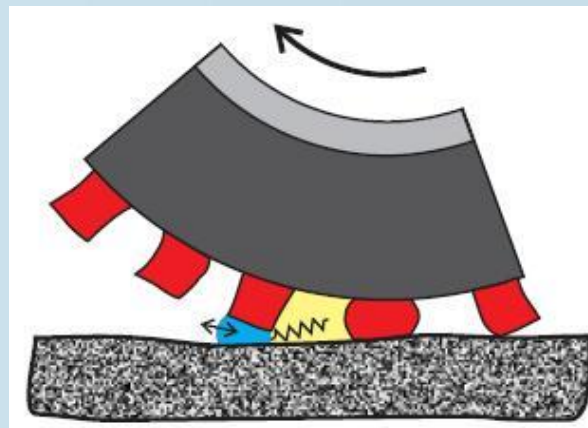
Air turbulence



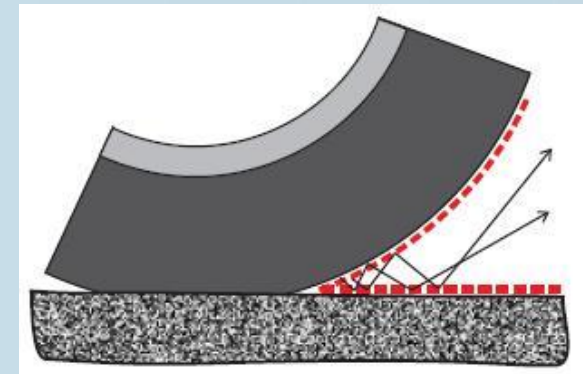
Air pumping



Pipe resonances



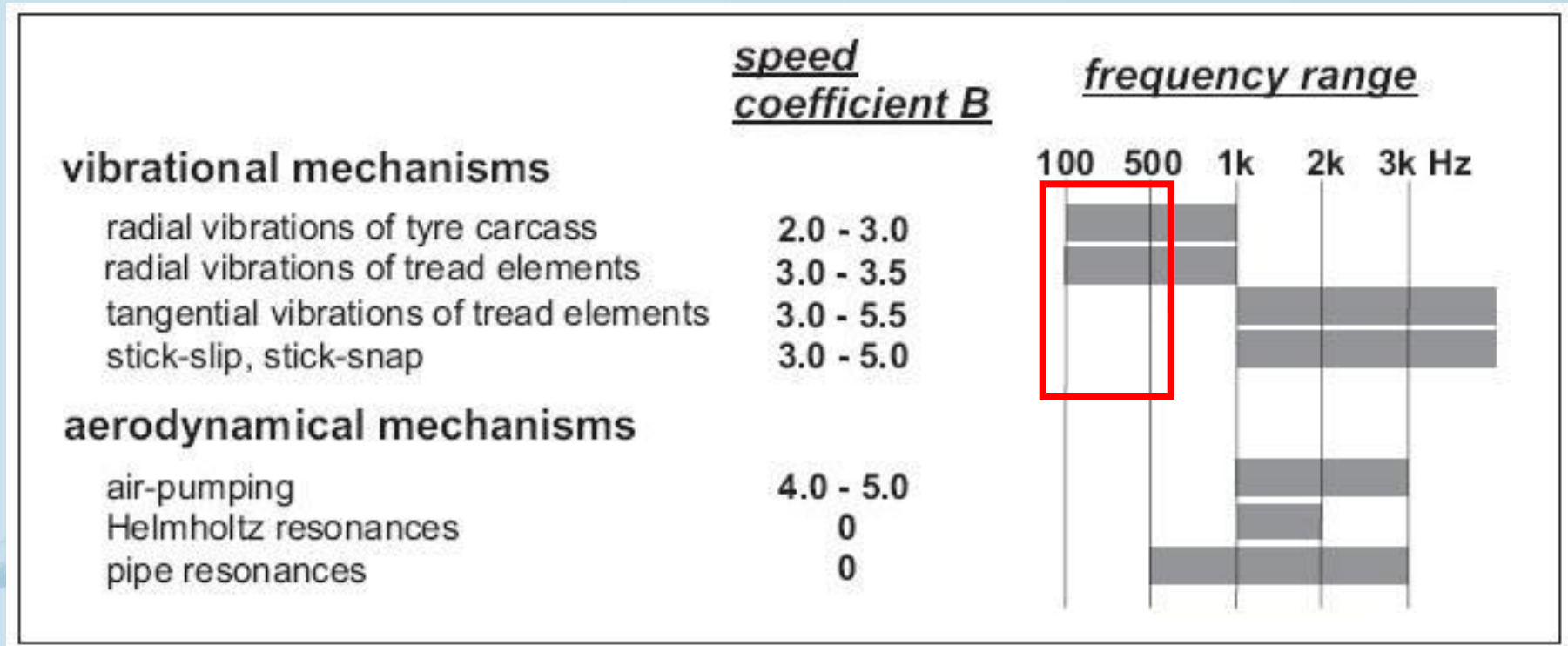
Helmholtz resonator



Horn amplification

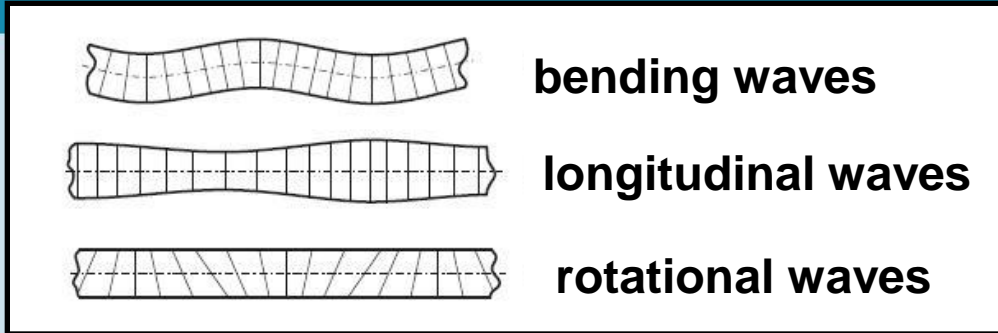


# Tyre/road noise



structure-borne tyre/road noise

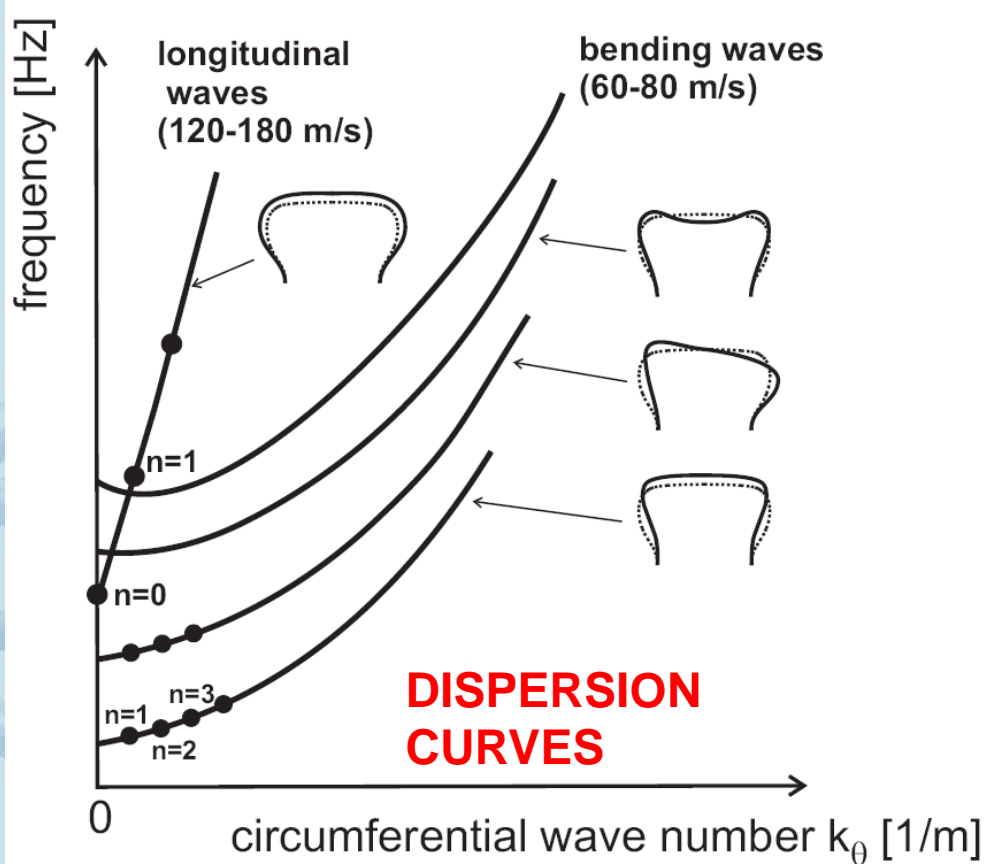
# Tyre dynamics



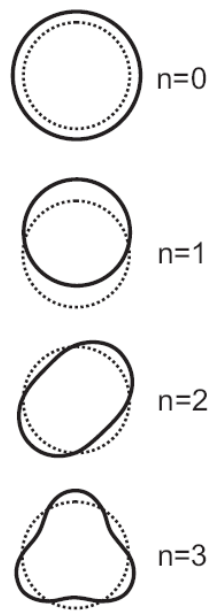
$$y = Ae^{-\alpha x} e^{j(\beta x - \omega t)}$$

wave number  $k = \beta + j\alpha$   $[m^{-1}]$

wavelength  $\lambda = \frac{2\pi}{k}$   $[m]$



circumferential mode number



**Resonance condition:**

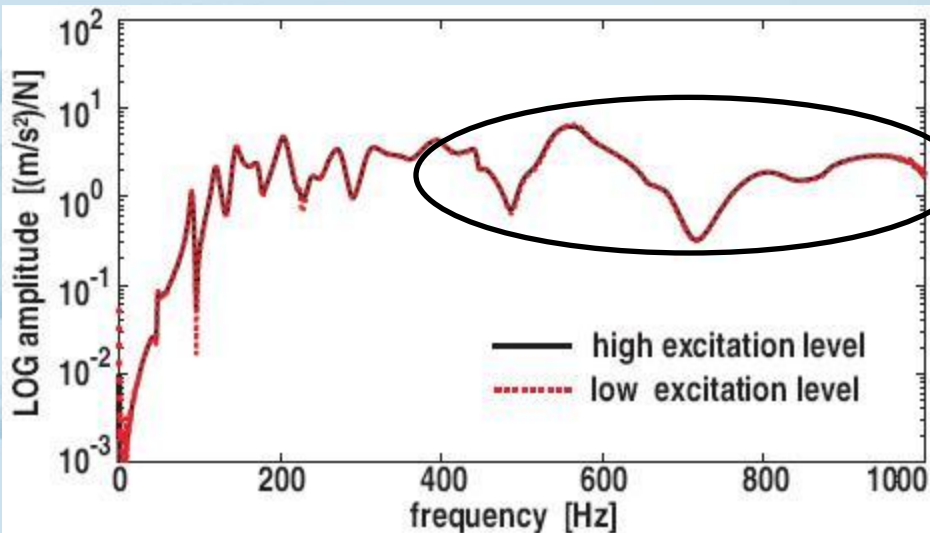
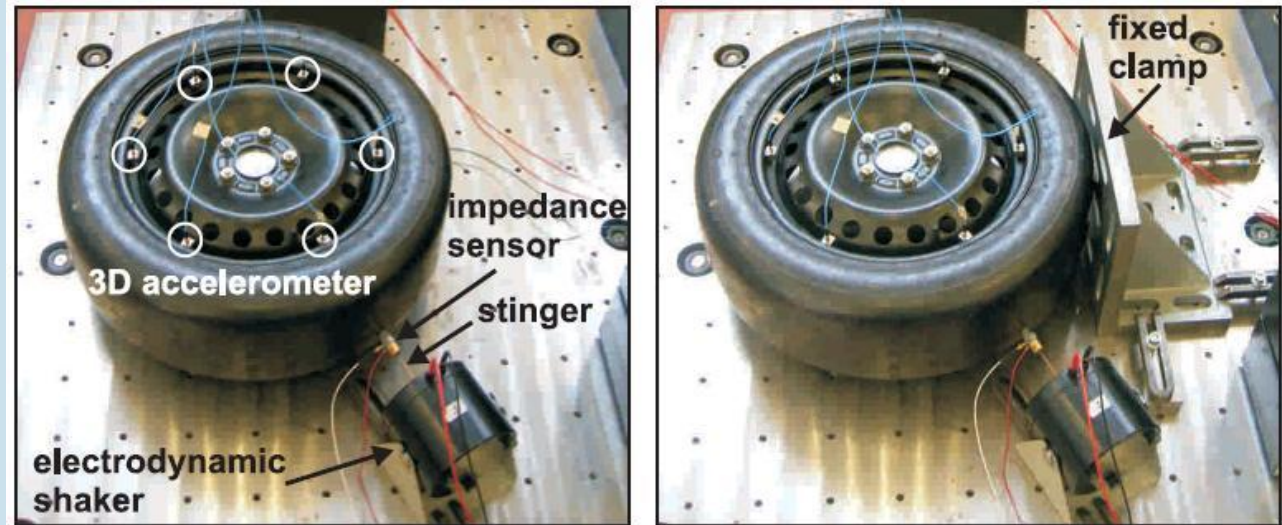
$$2\pi R = n\lambda \quad [m]$$

or

$$k_\theta = n/R \quad [m^{-1}]$$

# Experimental modal analysis

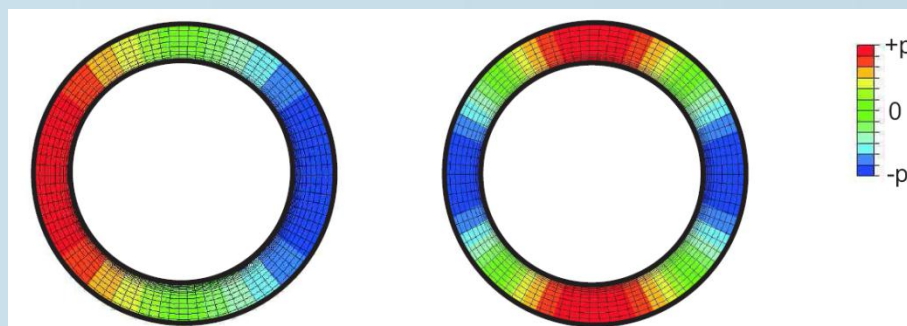
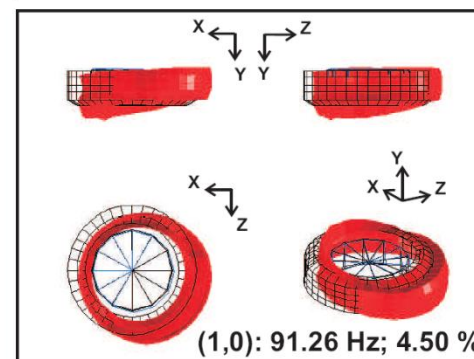
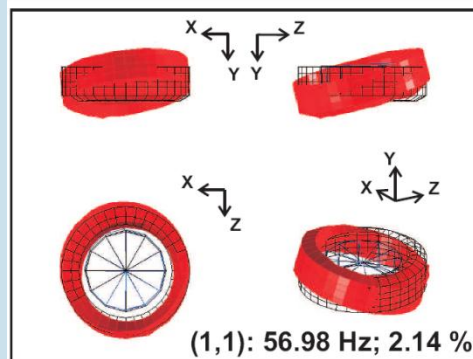
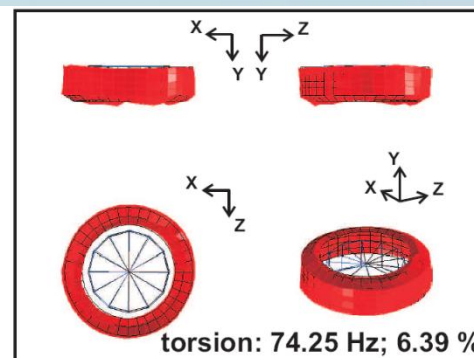
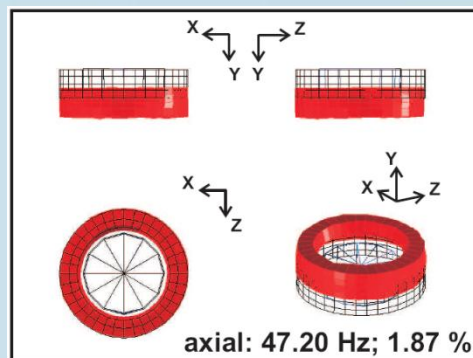
- Experimental analysis of the dynamic behaviour of a *non-rolling* tyre.



High modal density (2 modes/Hz)  
High damping of structural waves.

# Unloaded tyre

mode	freq. [Hz]	$\xi$ [%]
axial	47.20	1.87
torsional	74.25	6.39
(1,1)	56.98	2.14
(1,0)	91.26	4.50
(2,0)	118.55	3.26
(2,1)	89.37	2.07
(3,0)	141.97	2.78
(3,1)	171.91	2.73
(4,0)	169.96	3.01
(4,1)	222.88	3.57
(5,0)	201.79	2.41
(5,1)	252.63	4.13
(6,0)	233.75	2.82
(6,1)	279.90	3.65
(7,0)	268.96	2.98
(7,1)	*	*
(8,0)	306.04	3.18
(8,1)	*	*
1st acoustic	225.75	0.25
2nd acoustic	445.08	0.64
1st rim bending	187.22	2.12
rim pitch	189.89	0.69
rim axial	325.82	0.52
2nd rim bending	345.60	0.98



225.75 Hz; 0.25%

445.08 Hz; 0.64%



# Unloaded tyre

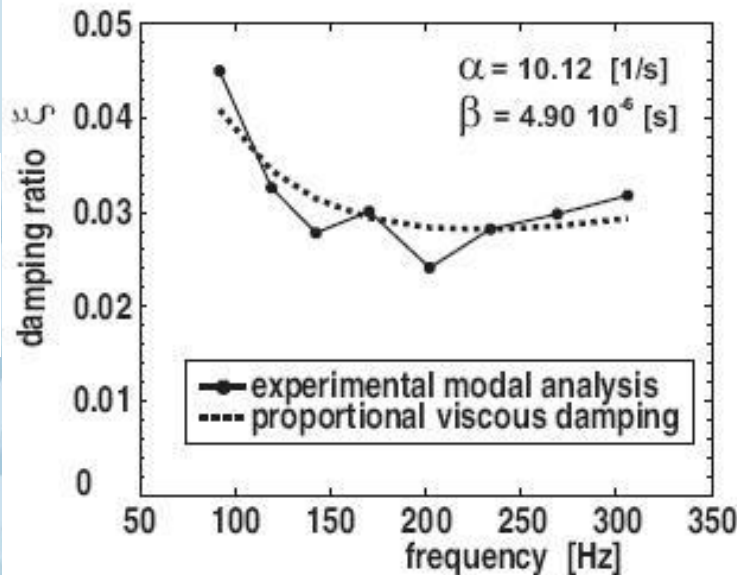
- Tyre damping

$$[C] = \alpha[M] + \beta[K]$$

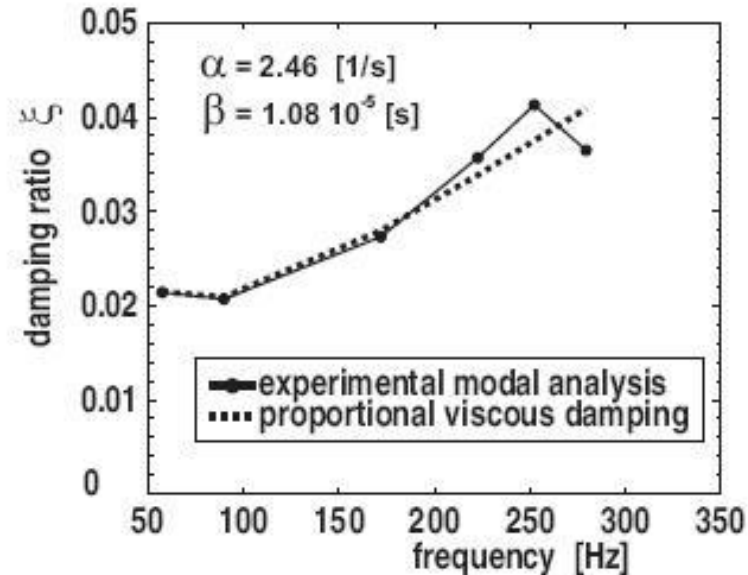
proportional viscous damping

$$\xi_r = \frac{\alpha}{2 \Omega_r} + \frac{\beta \Omega_r}{2} \quad \text{with} \quad \Omega_r = \frac{\omega_r}{\sqrt{1 - \xi_r^2}}$$

A more complex damping model is required.



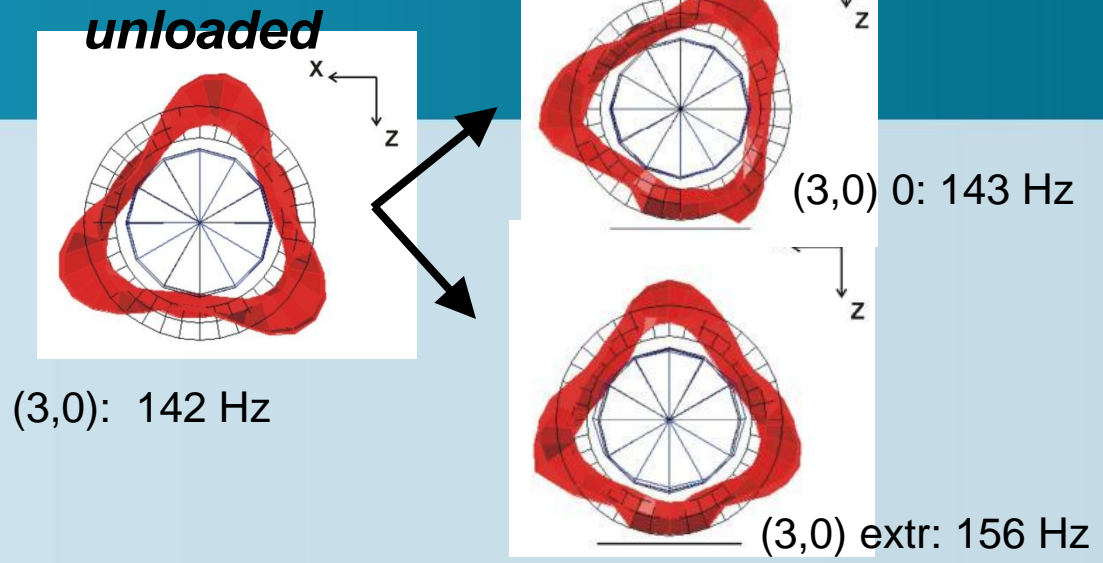
(a) (n,0) modes



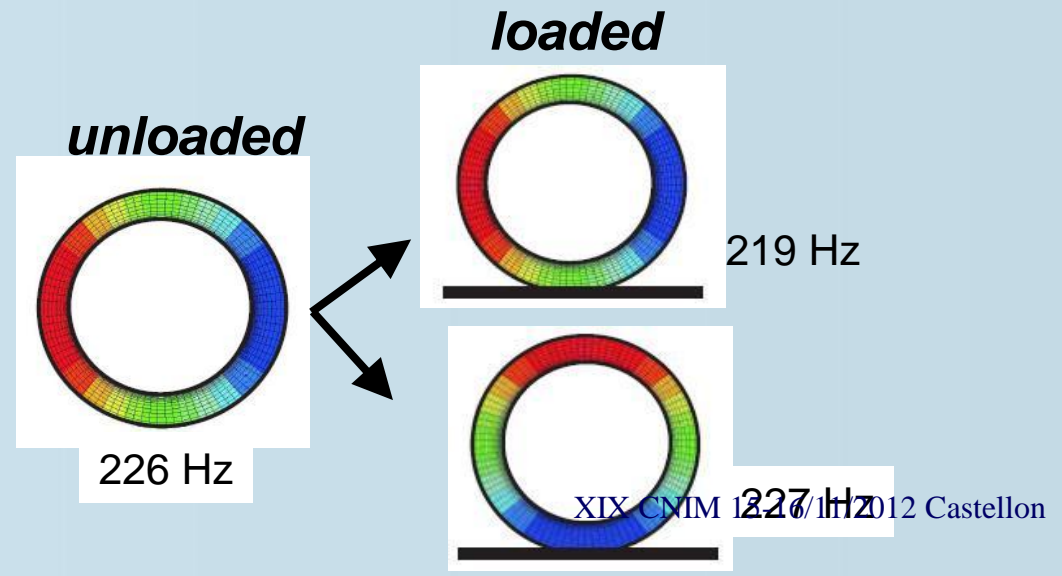
(b) (n,1) modes

# Loaded tyre

mode	no spindle rotation	
	freq. [Hz]	$\xi$ [%]
axial	/	/
torsional	/	/
(1,1) hor.	51.85	1.97
(1,1) vert.	64.75	2.48
(1,0) hor.	82.15	5.42
(1,0) vert.	98.22	4.09
(2,0) 0	117.94	3.43
(2,0) extr.	126.74	3.18
(2,1) 0	102.10	2.78
(2,1) extr.	86.99	2.46
(3,0) 0	142.56	2.87
(3,0) extr.	156.03	2.75
(3,1) 0	*	*
(3,1) extr.	162.36	2.78
(4,0) 0	172.66	2.89
(4,0) extr.	189.10	2.86
(4,1) 0	236.63	2.38
(4,1) extr.	225.23	1.44
(5,0) 0	205.11	3.10
(5,0) extr.	222.14	2.79
(5,1) 0	*	*
(5,1) extr.	258.03	3.47
(6,0) 0	242.34	2.75
(6,0) extr.	*	*
1st acoustic hor.	219.02	0.69
1st acoustic vert.	227.35	0.43
1st rim bending	183.13	2.53
rim pitch	189.62	1.15
rim axial	326.33	0.62

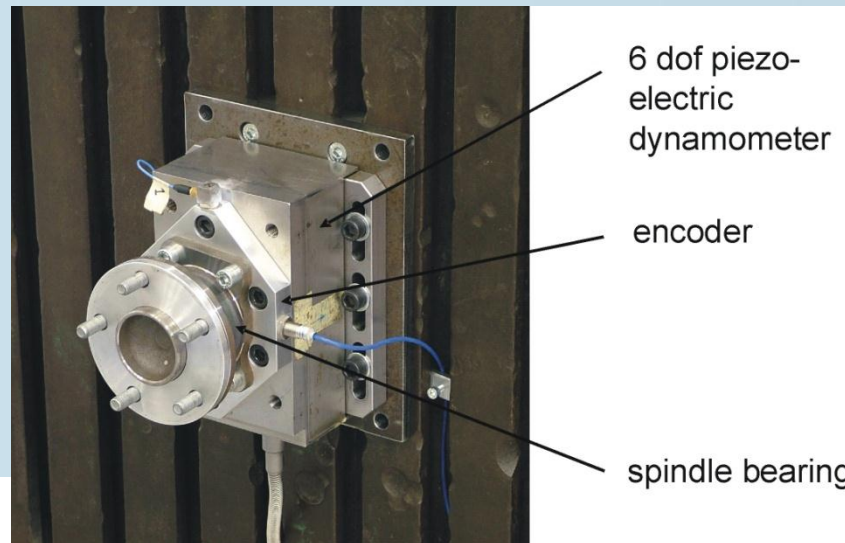
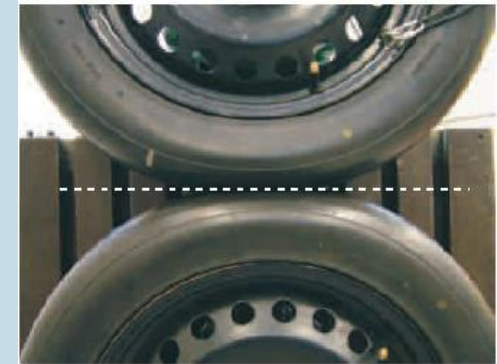
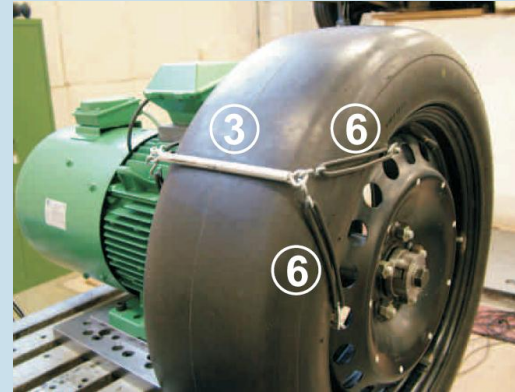
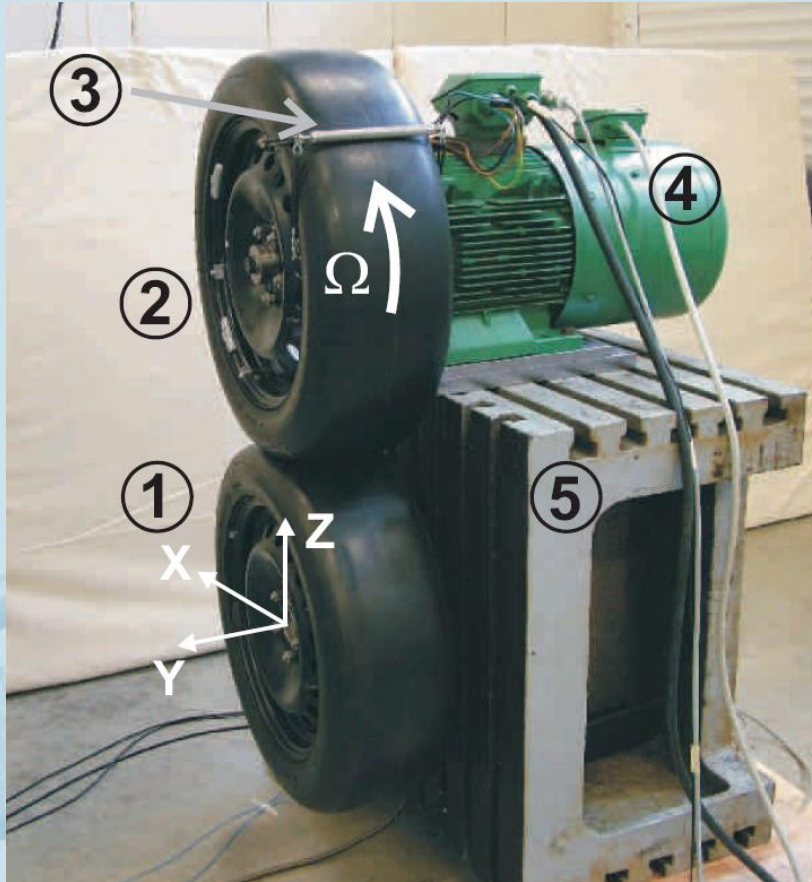
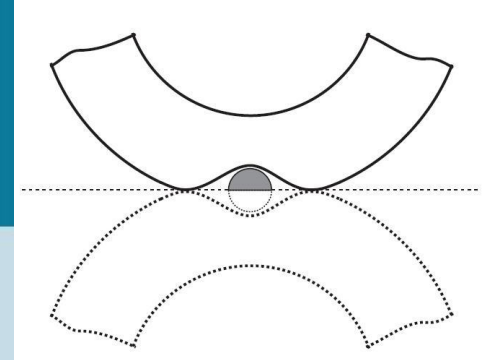


**Double poles** of unloaded tyre split up due to non-axisymmetry of loaded tyre.



# Tyre-on-tyre test setup

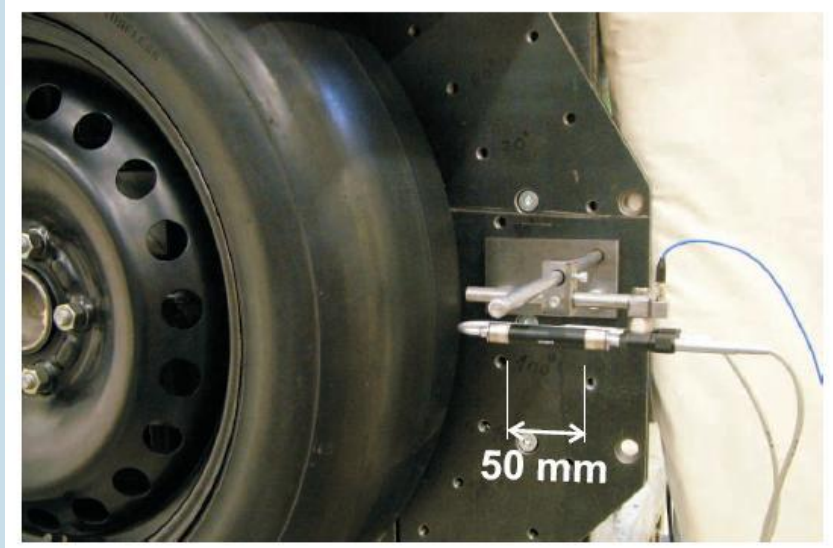
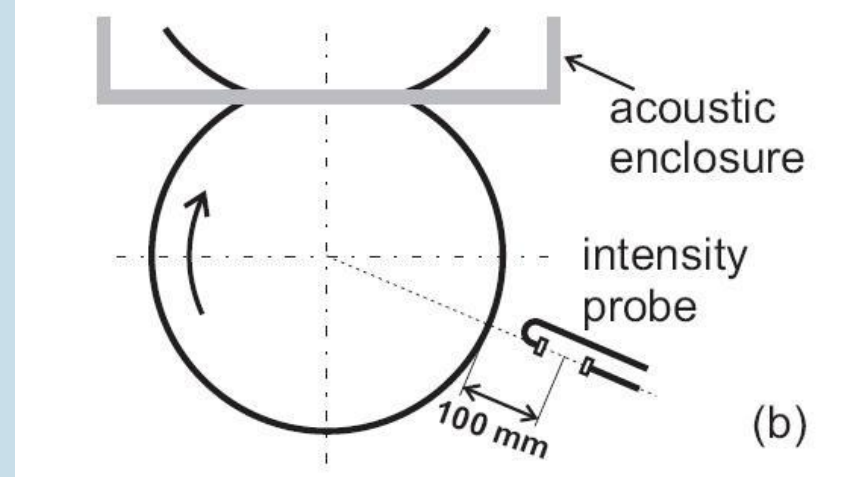
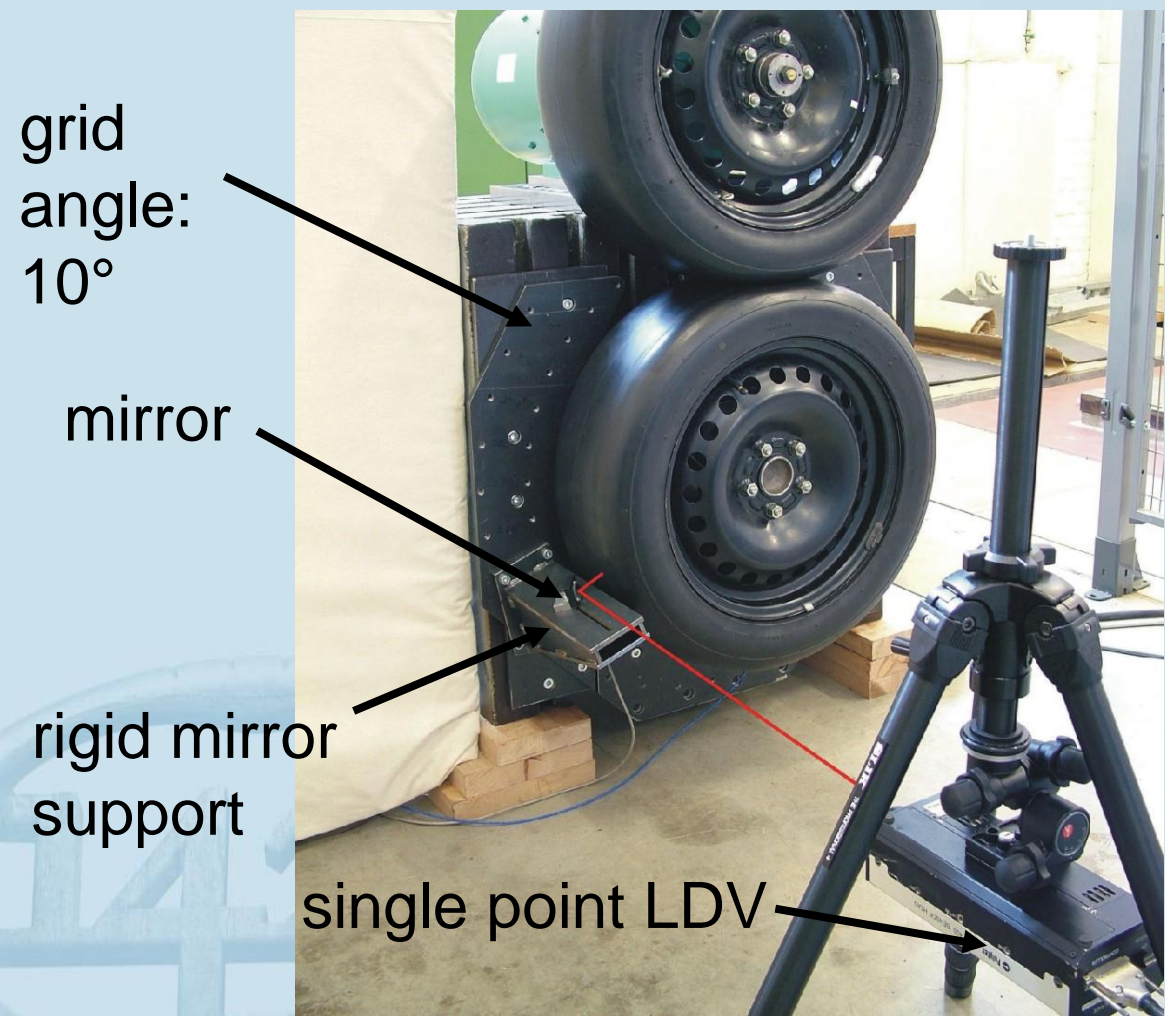
- 2 identical tyres



- |                       |                                   |
|-----------------------|-----------------------------------|
| <b>1: test tyre</b>   | <b>4: electric motor</b>          |
| <b>2: driven tyre</b> | <b>5: cast iron block</b>         |
| <b>3: cleat</b>       | <b>6: flexible cleat fixation</b> |



# Tyre-on-tyre test setup

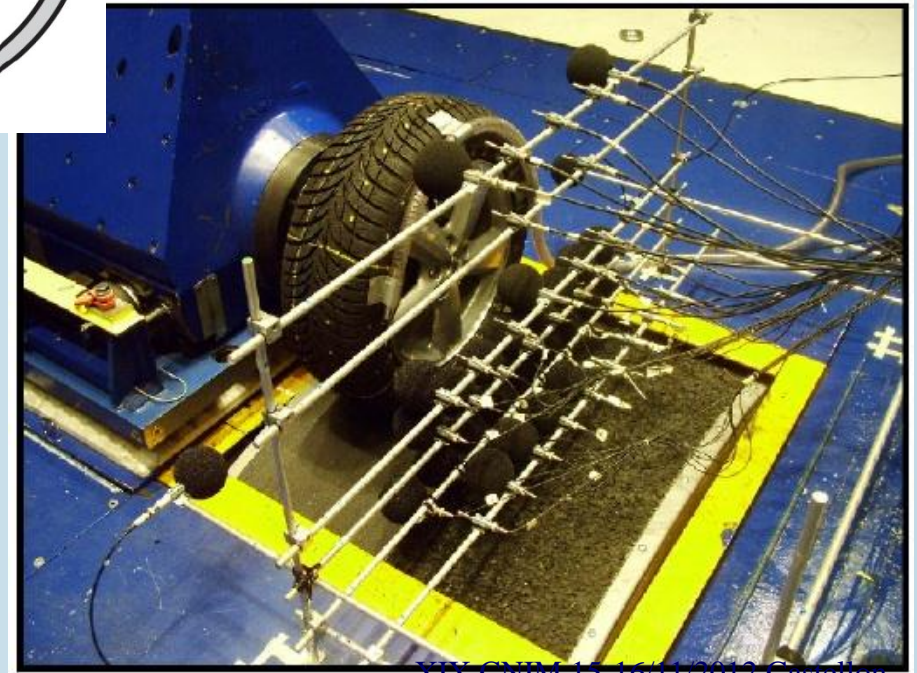
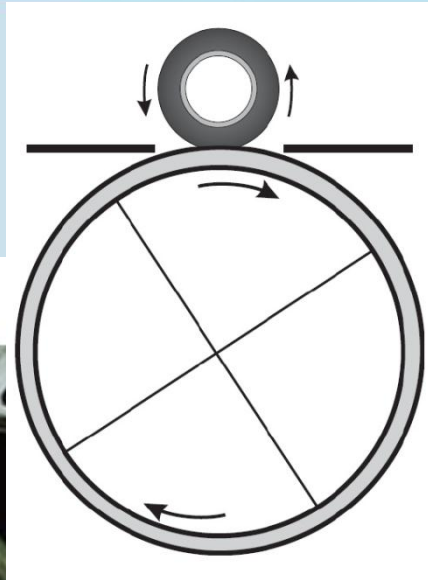
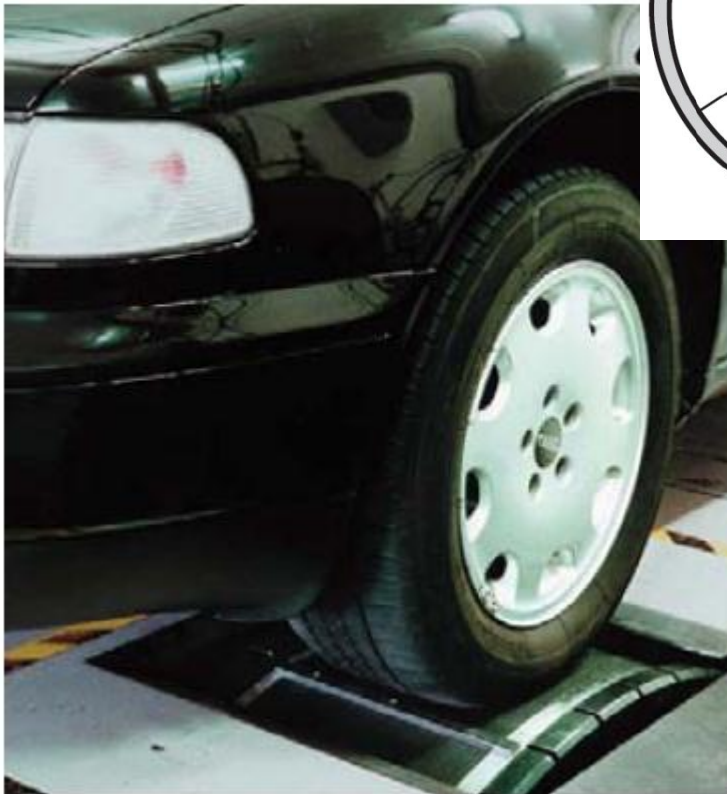


**Measurements relative to *FIXED REFERENCE FRAME***



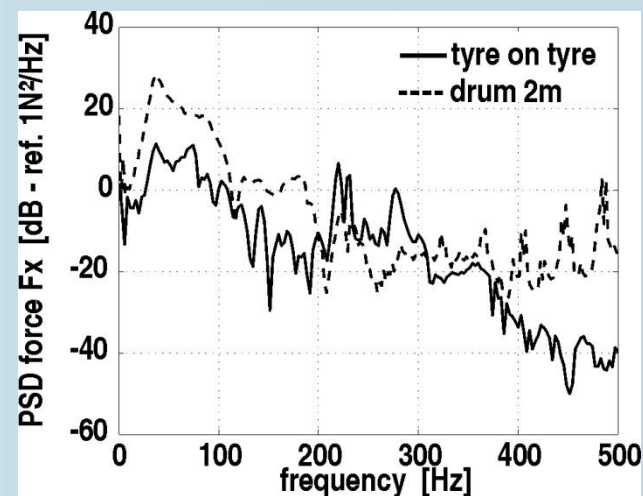
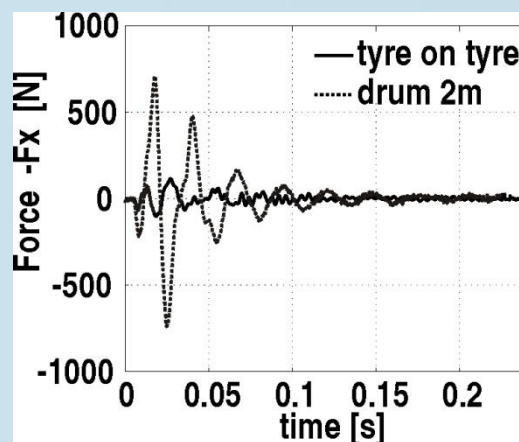
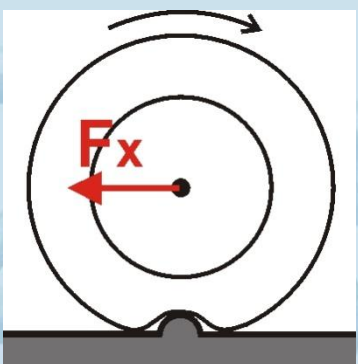
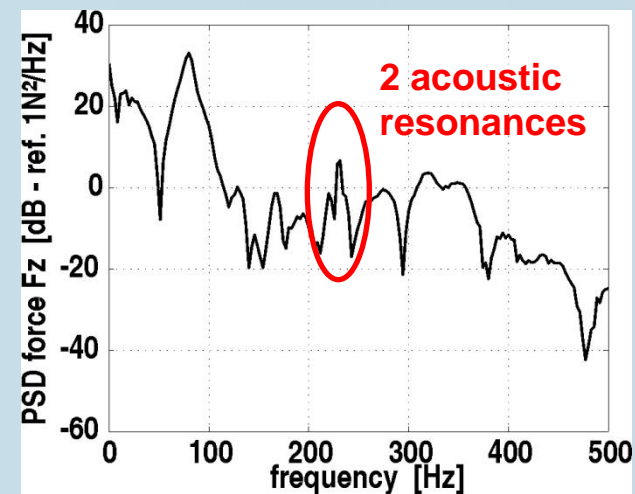
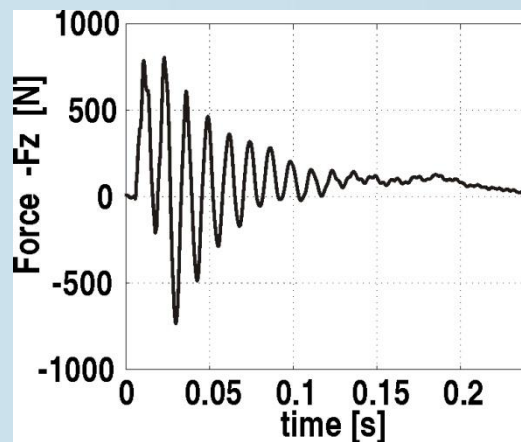
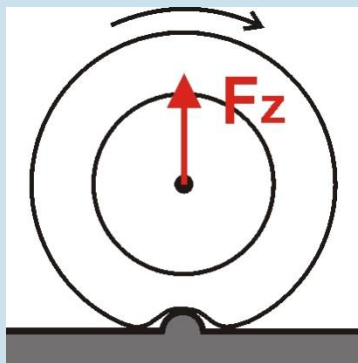
# Test Methods: Drum Tests

*Rolling rig for mechanical  
and acoustic comfort tests*

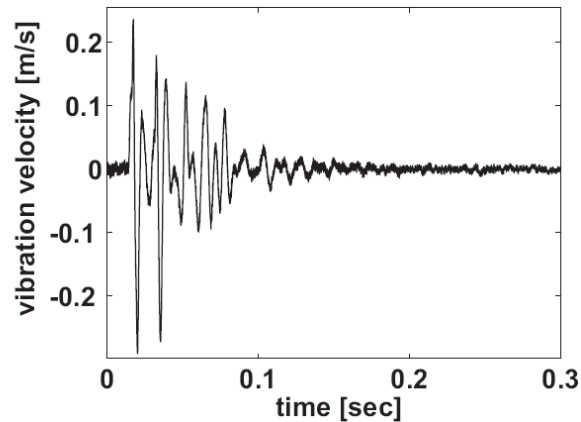
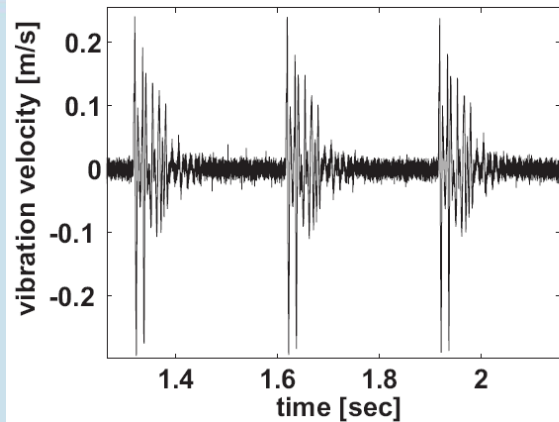


# Dynamic spindle forces

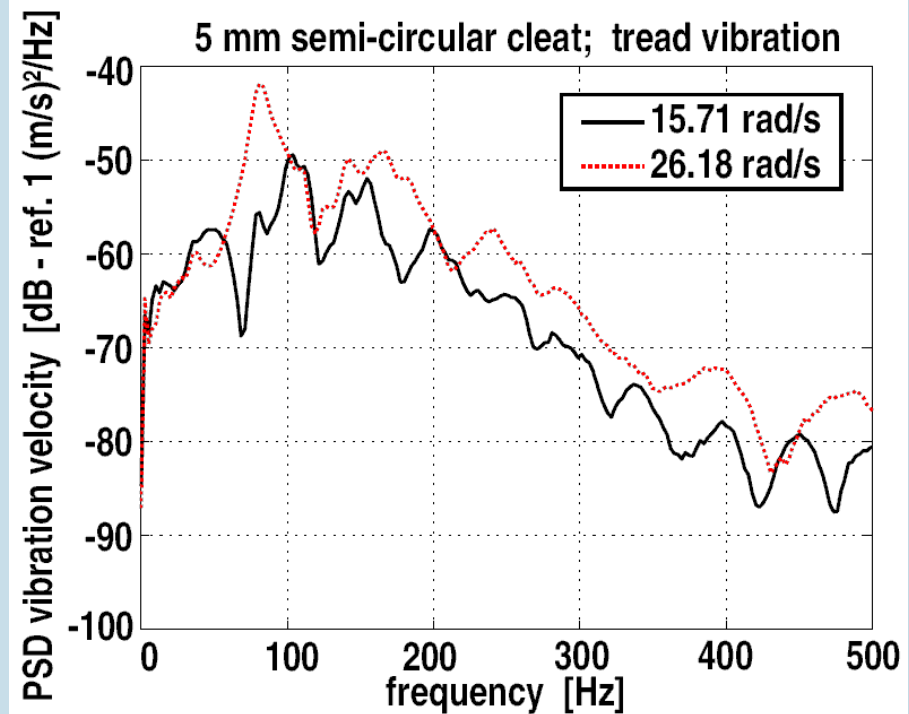
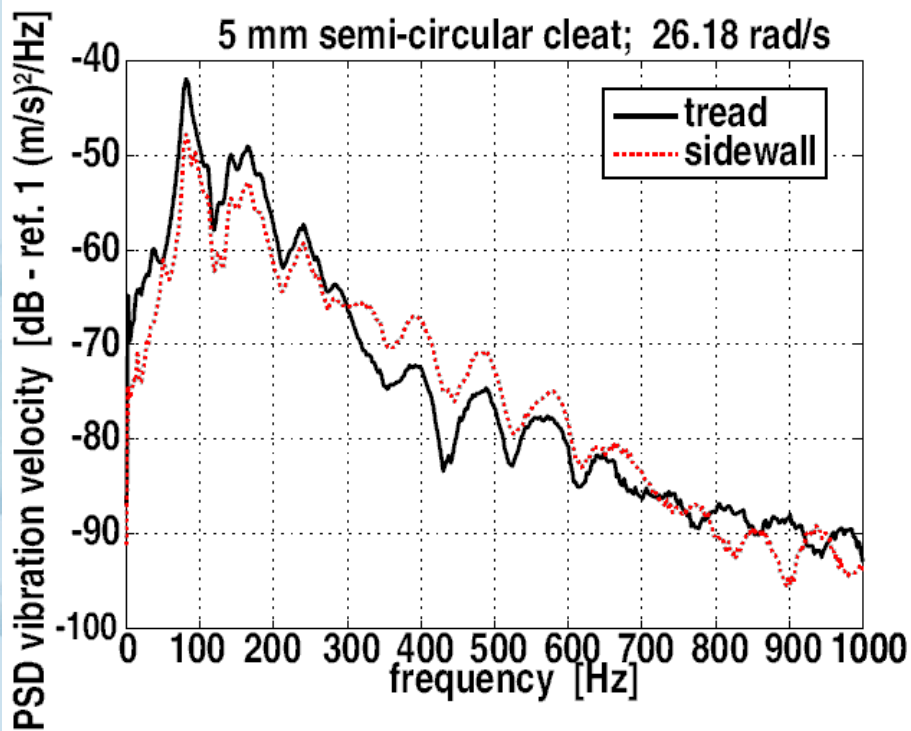
- 5 mm semi-circular cleat; 28 km/h; 2.2 bar



# Rolling tyre vibrations



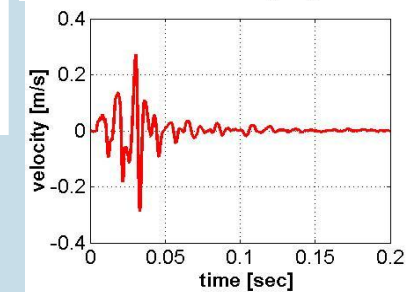
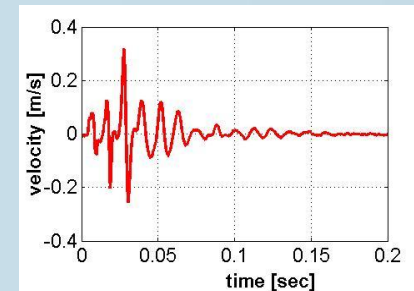
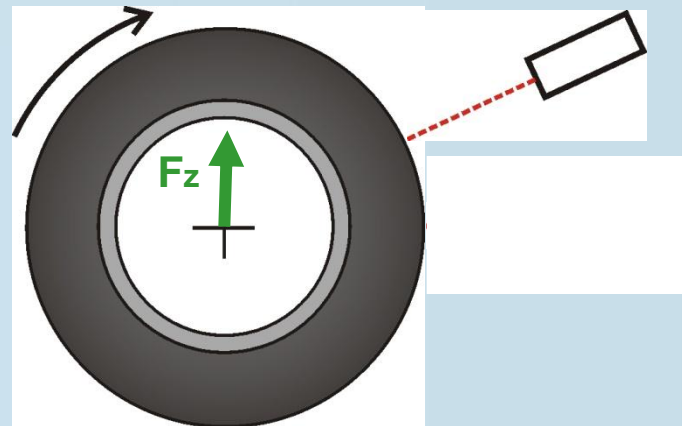
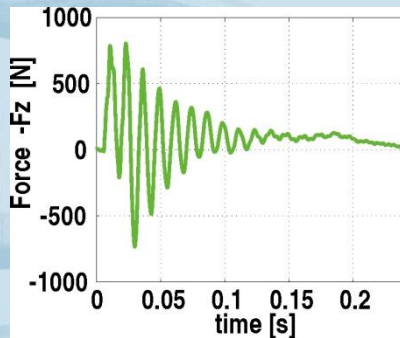
**time-averaged  
vibration signal**



# Operational Modal Analysis

- Excitation force of rolling tyre is difficult to measure.
- **Output-only method** → Polymax method applied to auto- and cross-power spectral density functions
- ***maintain phase relation*** between different response measurements:

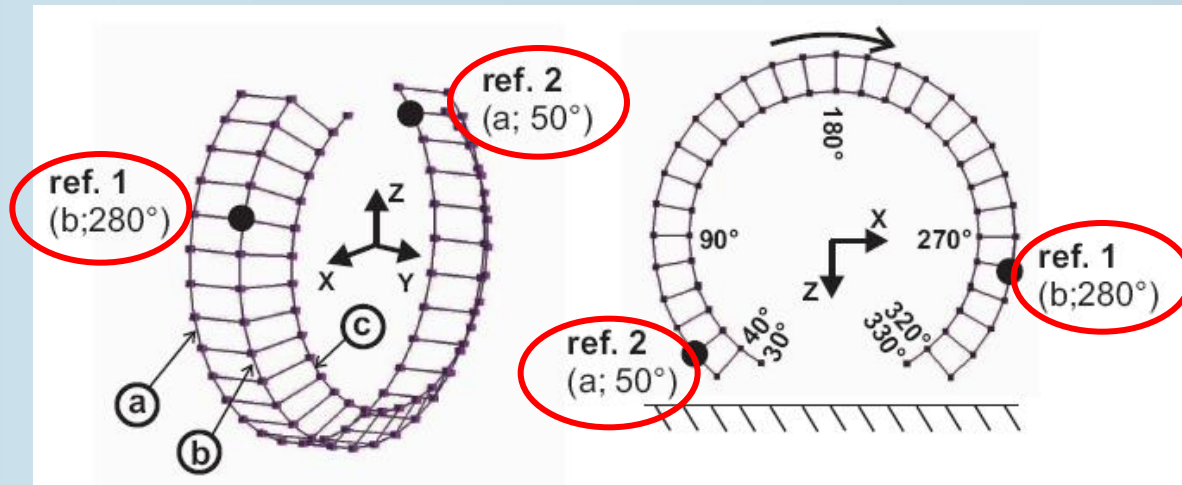
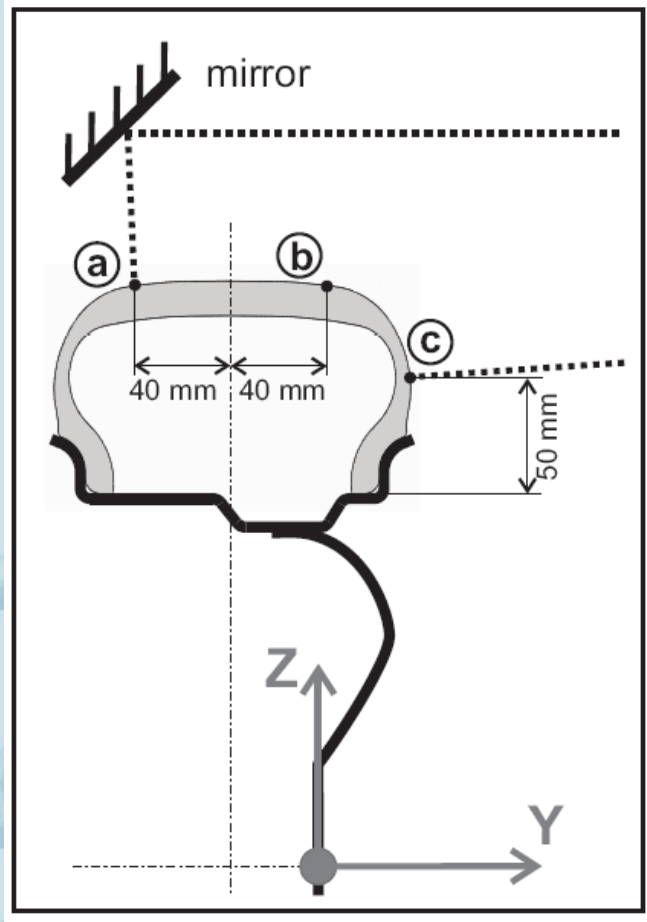
**time reference**  
(synchronization relative to excitation)



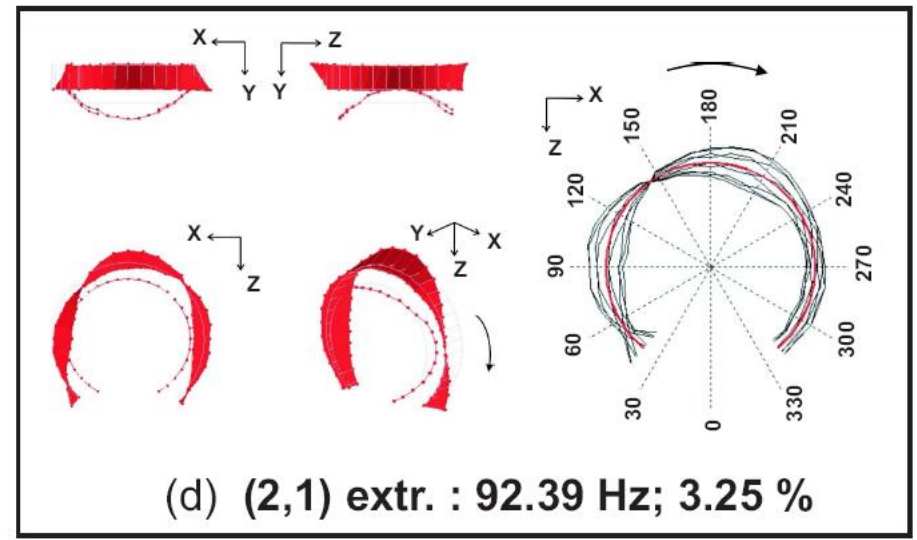
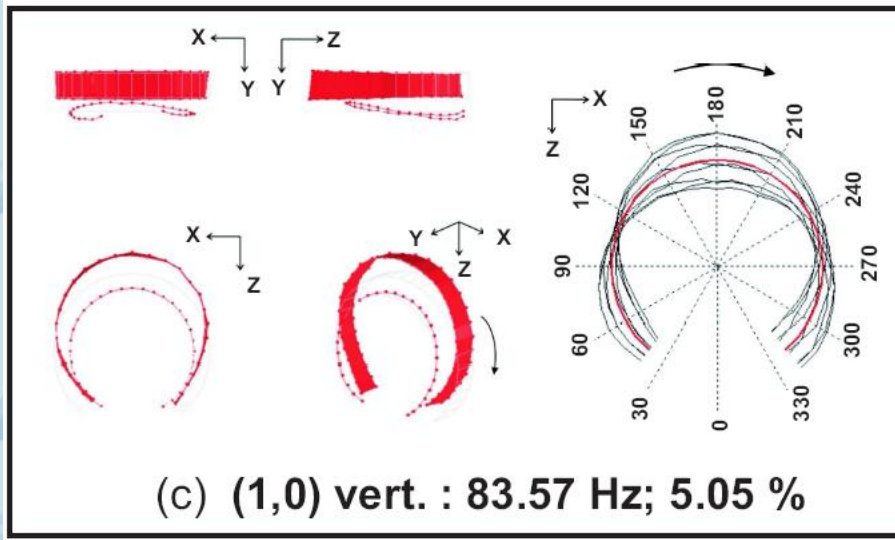
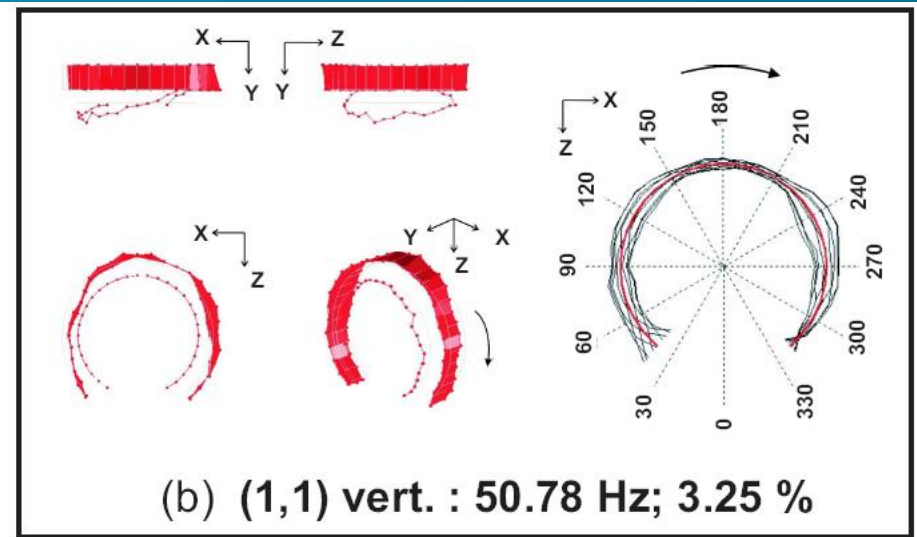
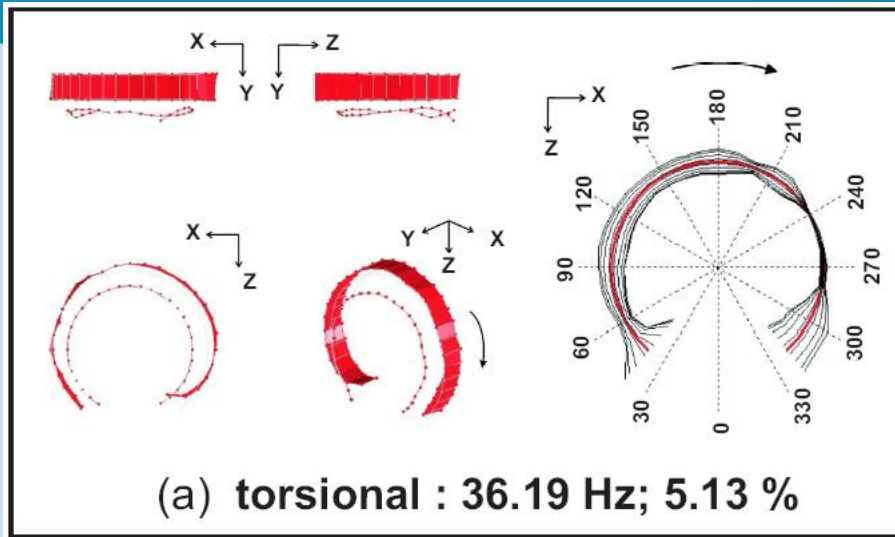


# Operational Modal Analysis

- Measurement geometry

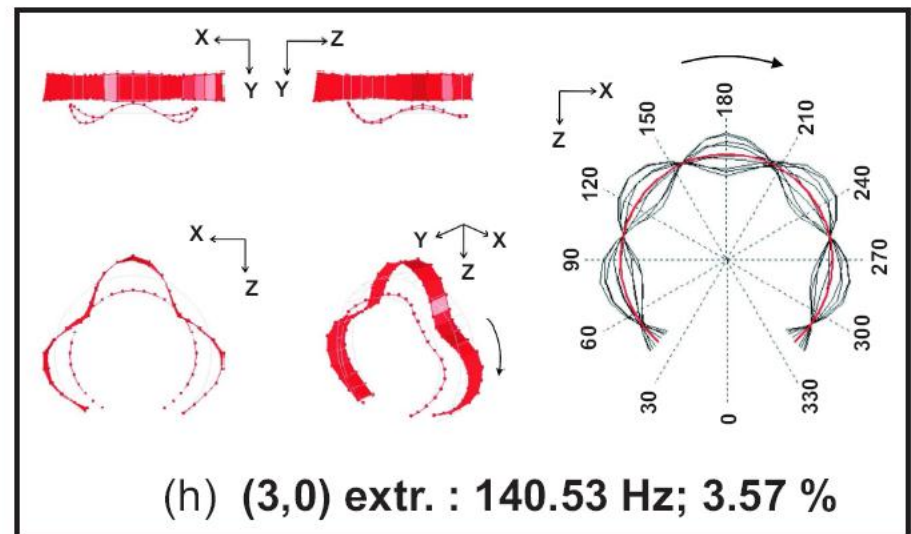
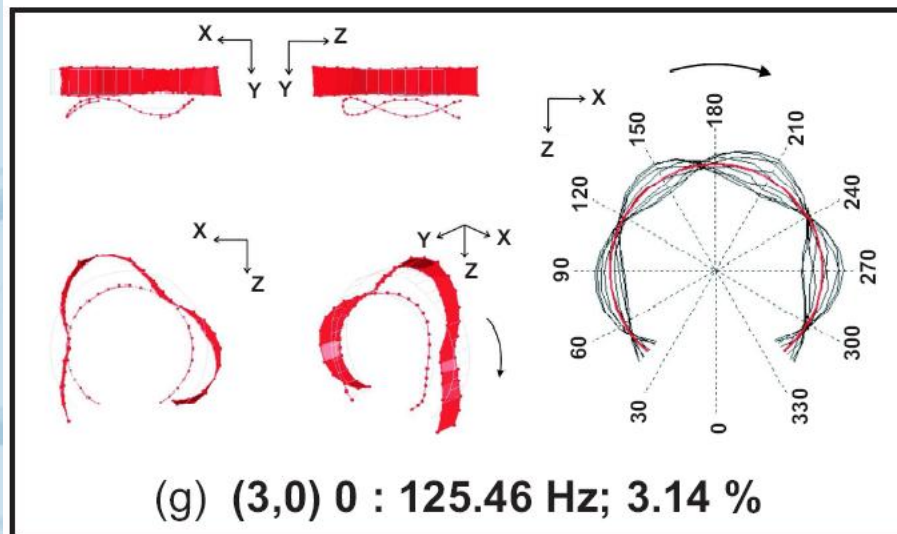
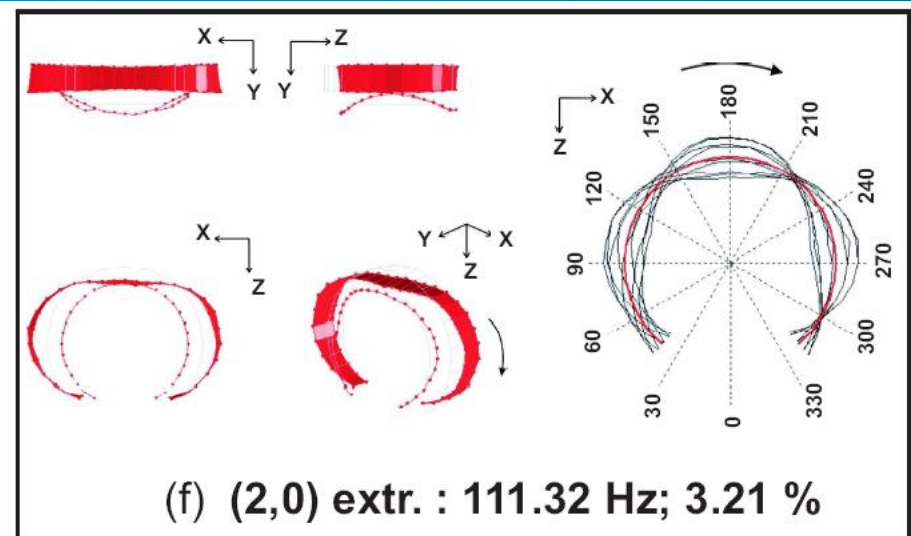
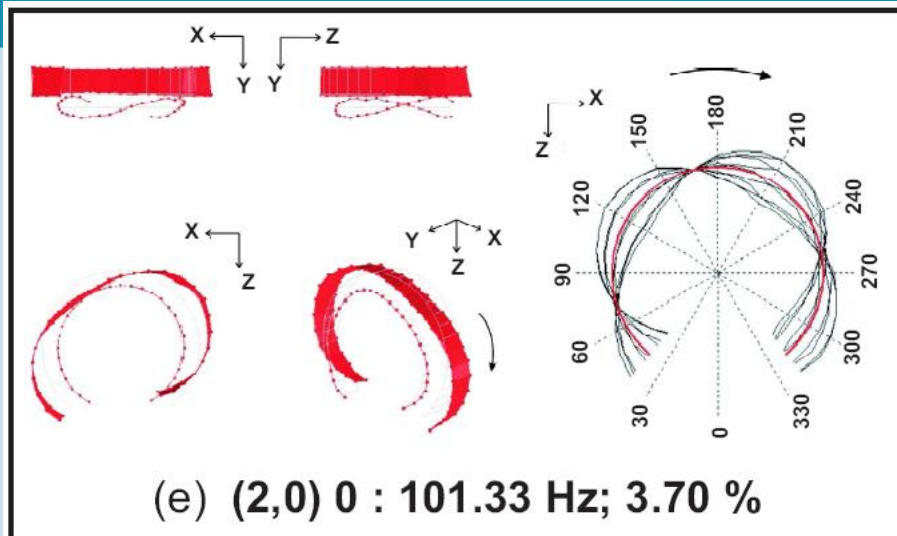


# Rolling tyre modal parameters (26.2rad/s)



modes relative to FIXED ref. system

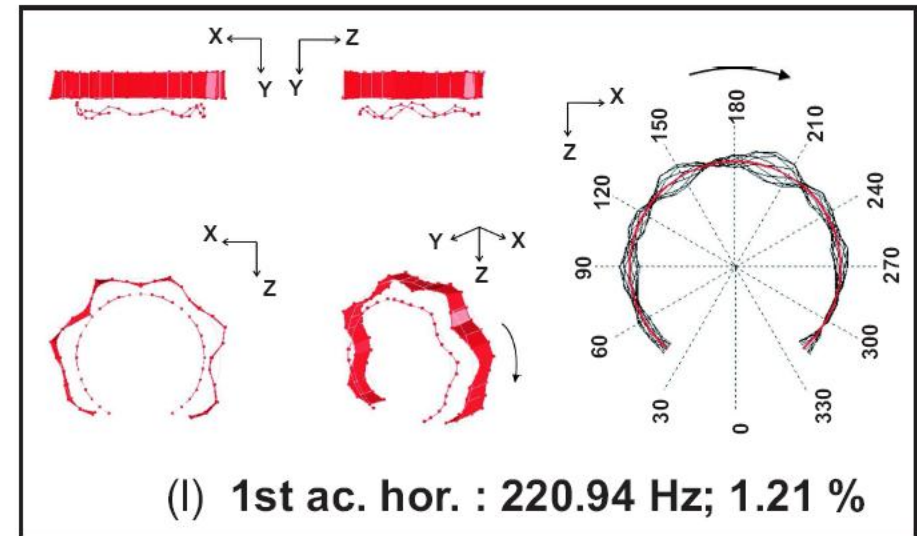
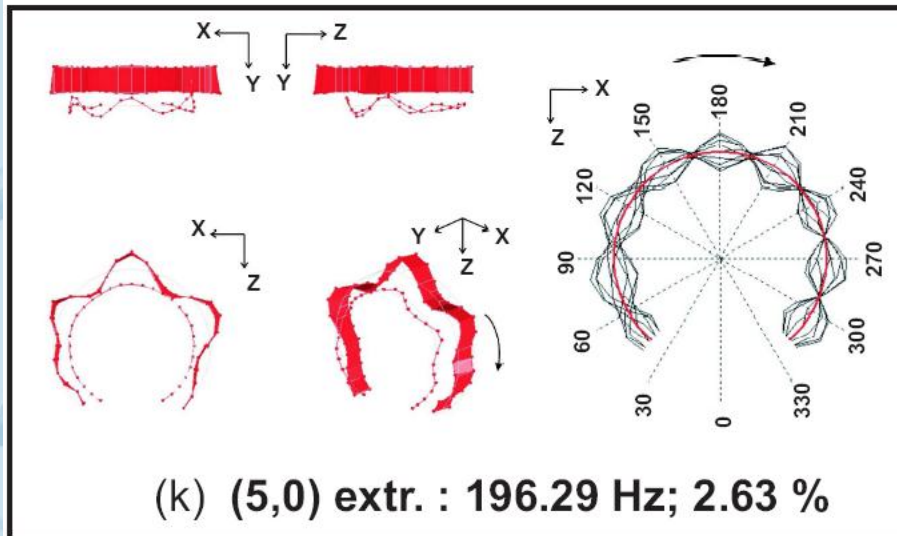
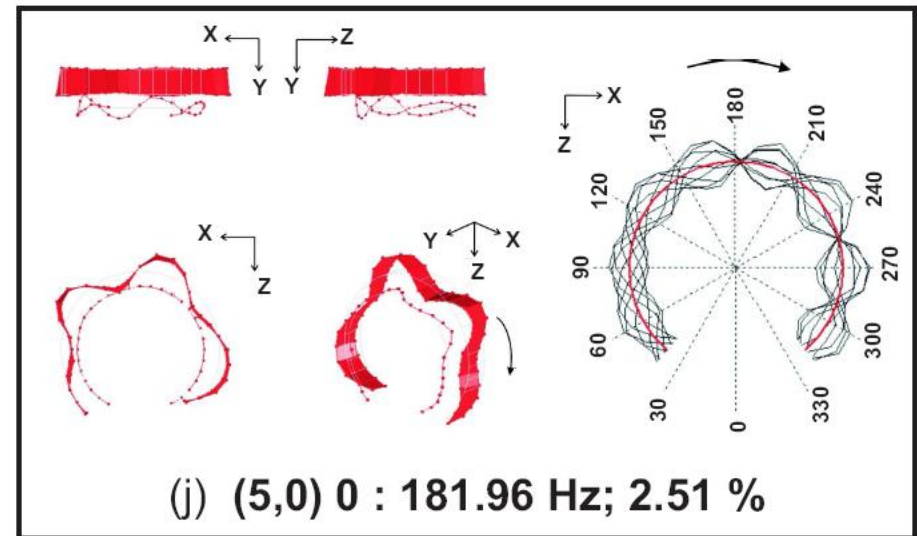
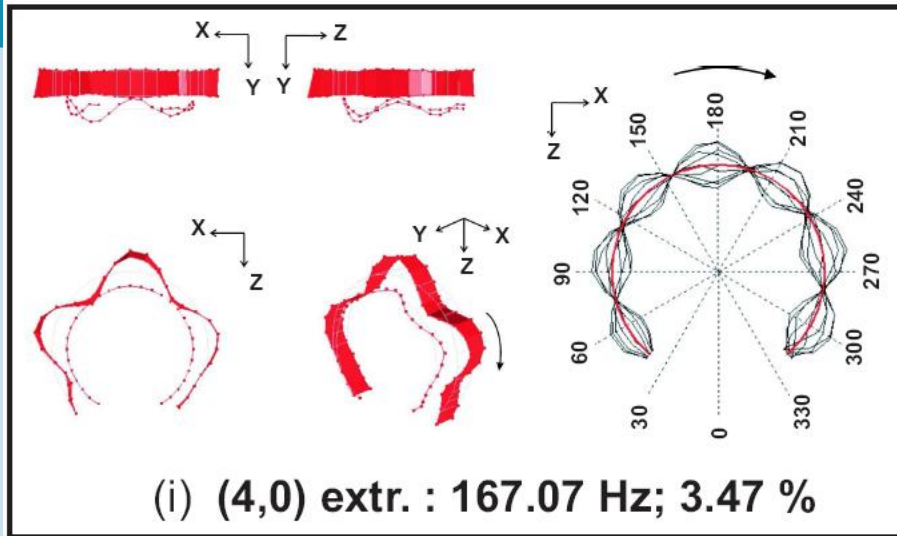
# Rolling tyre modal parameters (26.2rad/s)



modes relative to FIXED ref. system



# Rolling tyre modal parameters (26.2rad/s)

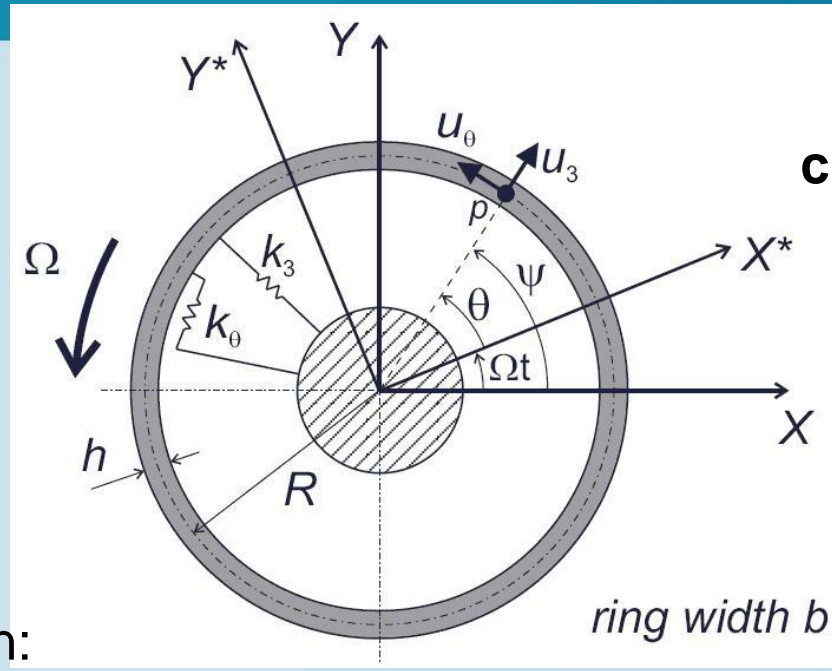


modes relative to FIXED ref. system



# Rotating flexible ring

$$\psi = \theta + \Omega t$$



co-rotating ref. system

fixed ref. system

Coriolis acceleration terms

Equations of motion:

$$\begin{aligned} & \frac{D}{R^4} \left( \frac{\partial^3 u_3}{\partial \theta^3} - \frac{\partial^2 u_\theta}{\partial \theta^2} \right) - \frac{K}{R^2} \left( \frac{\partial u_3}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial \theta^2} \right) + \frac{\sigma_{\theta\theta}^r h}{R^2} (u_\theta - 2 \frac{\partial u_3}{\partial \theta} - \frac{\partial^2 u_\theta}{\partial \theta^2}) \\ & + k_\theta u_\theta + \rho h \left( \frac{\partial^2 u_\theta}{\partial t^2} + 2\Omega \frac{\partial u_3}{\partial t} - \Omega^2 u_\theta \right) = q_\theta \\ & \frac{D}{R^4} \left( \frac{\partial^4 u_3}{\partial \theta^4} - \frac{\partial^3 u_\theta}{\partial \theta^3} \right) + \frac{K}{R^2} \left( \frac{\partial u_\theta}{\partial \theta} + u_3 \right) + \frac{\sigma_{\theta\theta}^r h}{R^2} \left( R + u_3 + 2 \frac{\partial u_\theta}{\partial \theta} - \frac{\partial^2 u_3}{\partial \theta^2} \right) \\ & + k_3 u_3 + \rho h \left( \frac{\partial^2 u_3}{\partial t^2} - 2\Omega \frac{\partial u_\theta}{\partial t} - \Omega^2 u_3 - R\Omega^2 \right) = q_3 \end{aligned}$$

# Rotating ring in **co-rotating** ref. system

$$\omega_{n1} = \frac{2n}{n^2 + 1}\Omega + \sqrt{\omega_{fn}^2 + \frac{n^2(n^2 - 1)^2}{(n^2 + 1)^2}\Omega^2} > 0$$

$$\omega_{n2} = \frac{2n}{n^2 + 1}\Omega - \sqrt{\omega_{fn}^2 + \frac{n^2(n^2 - 1)^2}{(n^2 + 1)^2}\Omega^2} < 0$$

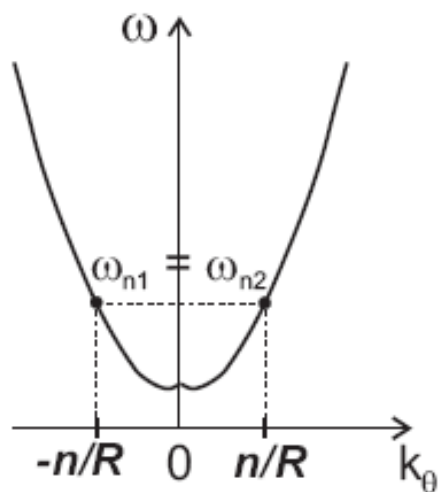
$$u_3(\theta, t) = A_n e^{j(n\theta + \omega_n t)}$$

$$u_\theta(\theta, t) = B_n e^{j(n\theta + \omega_n t)}$$

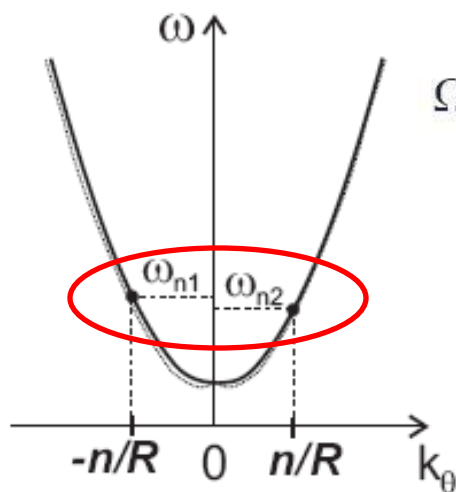
**mode shapes**

## natural frequencies

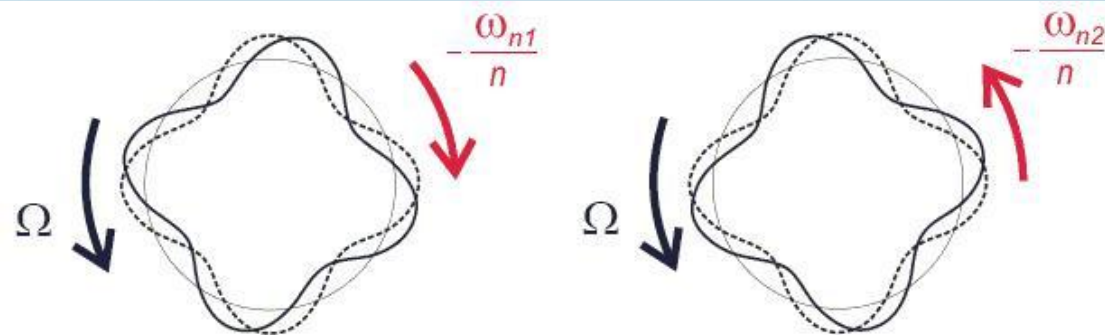
(n = circumferential mode number)



(a) 0 rad/s



(b) Ω rad/s; co-rotating



(a) mode at  $\omega_{n1}$

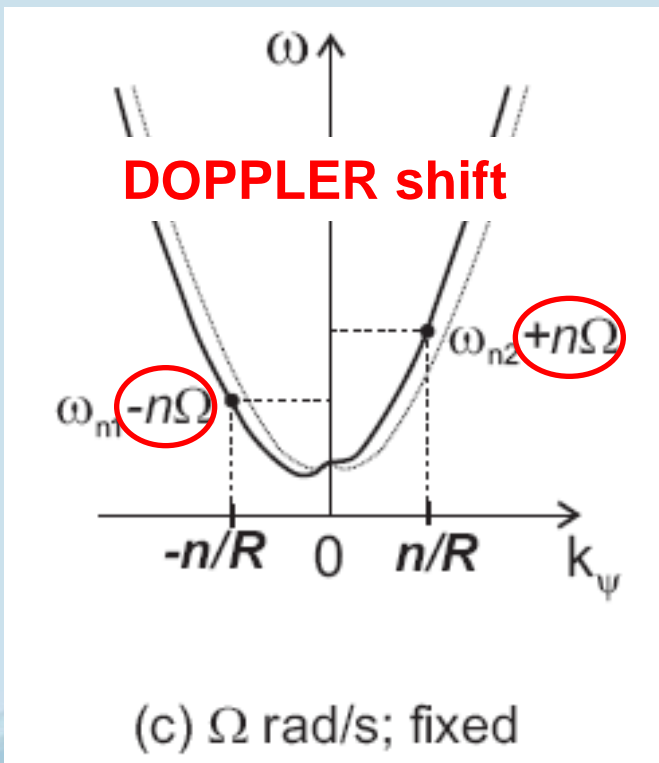
(b) mode at  $\omega_{n2}$

**backward  
travelling  
wave**

**forward  
travelling  
wave**

**BIFURCATION effect**

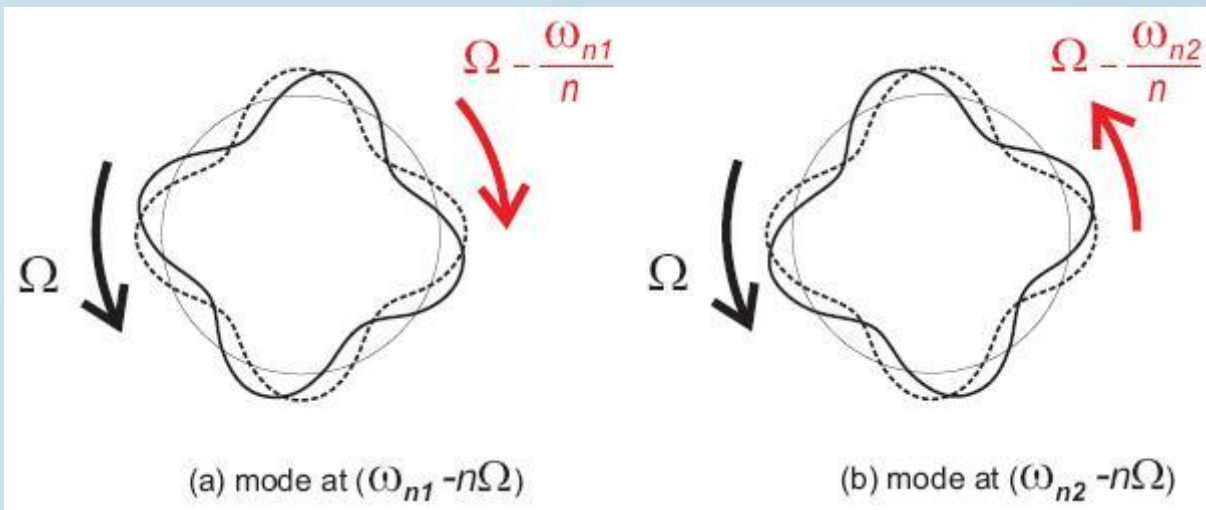
# Rotating ring in **fixed** ref. system



## mode shapes

$$u_z(\psi, t) = \sin(n\psi + (\omega_{nk} - n\Omega)t)$$

$$u_\theta(\psi, t) = -C_{nk} \cos(n\psi + (\omega_{nk} - n\Omega)t)$$



**backward  
travelling  
wave**

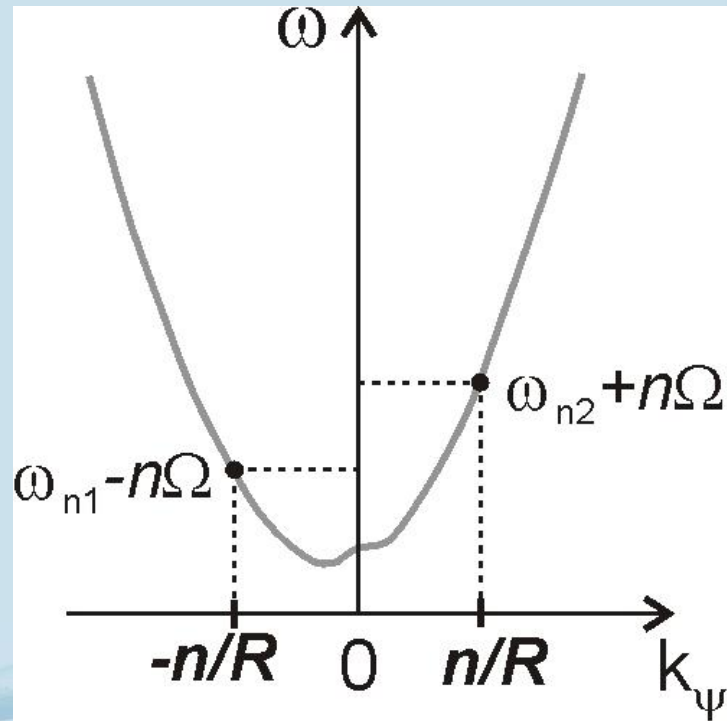
**forward  
travelling  
wave**

# Rolling tyre modal parameters

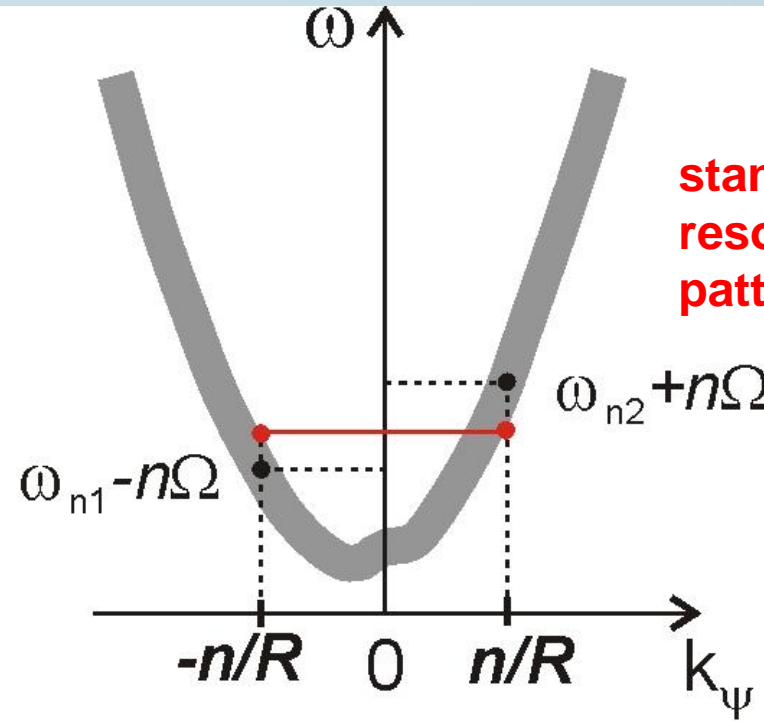
- **ANALYTICAL ROTATING RING**: a forward and backward travelling wave cannot interfere at a single natural frequency to form a standing wave pattern
    - at resonance: travelling wave deformation pattern
  - **EXPERIMENT**: standing wave patterns with respect to the fixed reference frame
    - Influence of:
      - damping
      - disturbed geometrical symmetry
- on rolling tyre dynamic behaviour is not yet fully understood.



# Influence of damping on dispersion curve



$\Omega$  rad/s; fixed  
**LOW DAMPING**



$\Omega$  rad/s; fixed  
**HIGH DAMPING**

standing wave  
resonance  
pattern

# Influence of rolling speed

mode	Frequency [Hz] / $\xi$ [%]		
	0 rad/s	15.71 rad/s	26.18 rad/s
torsional	<b>46.98</b> / 3.95	<b>36.76</b> / 3.29	<b>36.19</b> / 5.13
(1,1) hor.	<b>49.06</b> / 1.38	<b>45.05</b> / 2.46	*
(1,1) vert.	<b>58.58</b> / 2.26	<b>51.23</b> / 1.99	<b>50.78</b> / 3.25
(1,0) hor.	<b>111.96</b> / 3.97	*	*
(1,0) vert.	<b>95.94</b> / 3.78	<b>79.42</b> / 2.40	<b>83.57</b> / 5.05
(2,0) 0	<b>117.34</b> / 3.24	<b>101.94</b> / 4.05	<b>101.33</b> / 3.70
(2,0) extr.	<b>124.03</b> / 2.94	<b>113.13</b> / 3.37	<b>111.32</b> / 3.21
(2,1) 0	<b>100.15</b> / 2.72	*	*
(2,1) extr.	<b>85.22</b> / 2.36	<b>88.51</b> / 2.41	<b>92.39</b> / 3.25
(3,0) 0	<b>141.22</b> / 3.19	<b>127.61</b> / 2.49	<b>125.46</b> / 3.14
(3,0) extr.	<b>155.90</b> / 2.71	<b>141.27</b> / 3.11	<b>140.53</b> / 3.57
(3,1) 0	*	*	*
(3,1) extr.	<b>167.41</b> / 2.59	*	*
(4,0) 0	<b>171.18</b> / 2.68	<b>155.98</b> / 3.33	*
(4,0) extr.	<b>187.14</b> / 2.49	<b>169.58</b> / 1.95	<b>167.07</b> / 3.47
(4,1) 0	*	*	*
(4,1) extr.	<b>227.09</b> / 2.16	*	*
(5,0) 0	<b>203.39</b> / 2.89	<b>184.10</b> / 1.77	<b>181.96</b> / 2.51
(5,0) extr.	<b>219.63</b> / 2.32	<b>199.46</b> / 2.84	<b>196.29</b> / 2.63
1st acoustic hor.	<b>218.99</b> / 0.39	<b>222.56</b> / 0.34	<b>220.94</b> / 1.21
1st acoustic vert.	<b>226.23</b> / 0.31	<b>228.65</b> / 1.02	<b>228.38</b> / 0.28

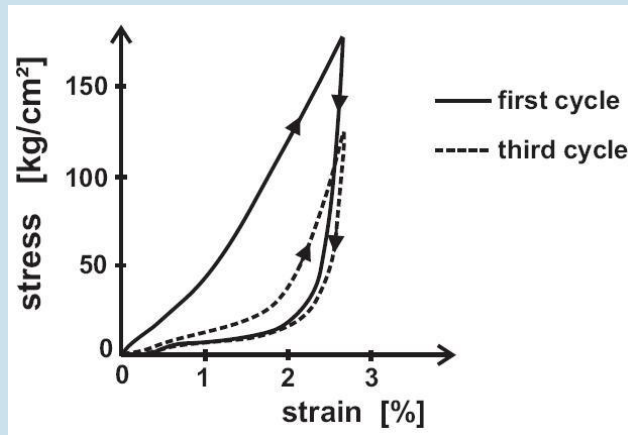
drop in resonance  
frequency as the tyre  
starts to roll

(n,0) modes: **-10.8 %**

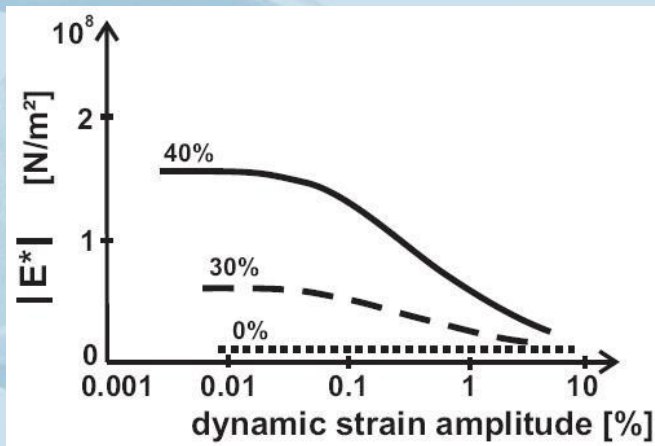
# Initial drop in resonance frequencies

- Drop in resonance frequencies as the tyre starts to roll:

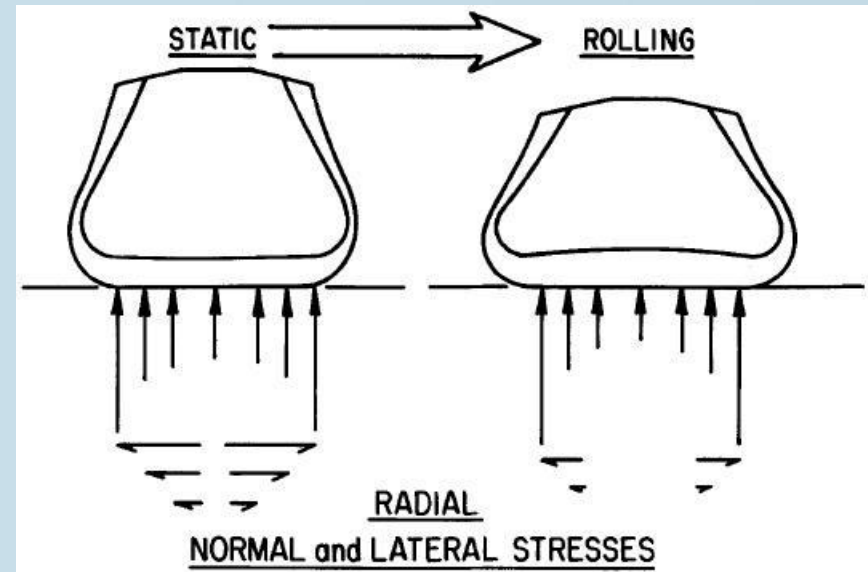
## 1) Mullins effect



## 2) Payne effect

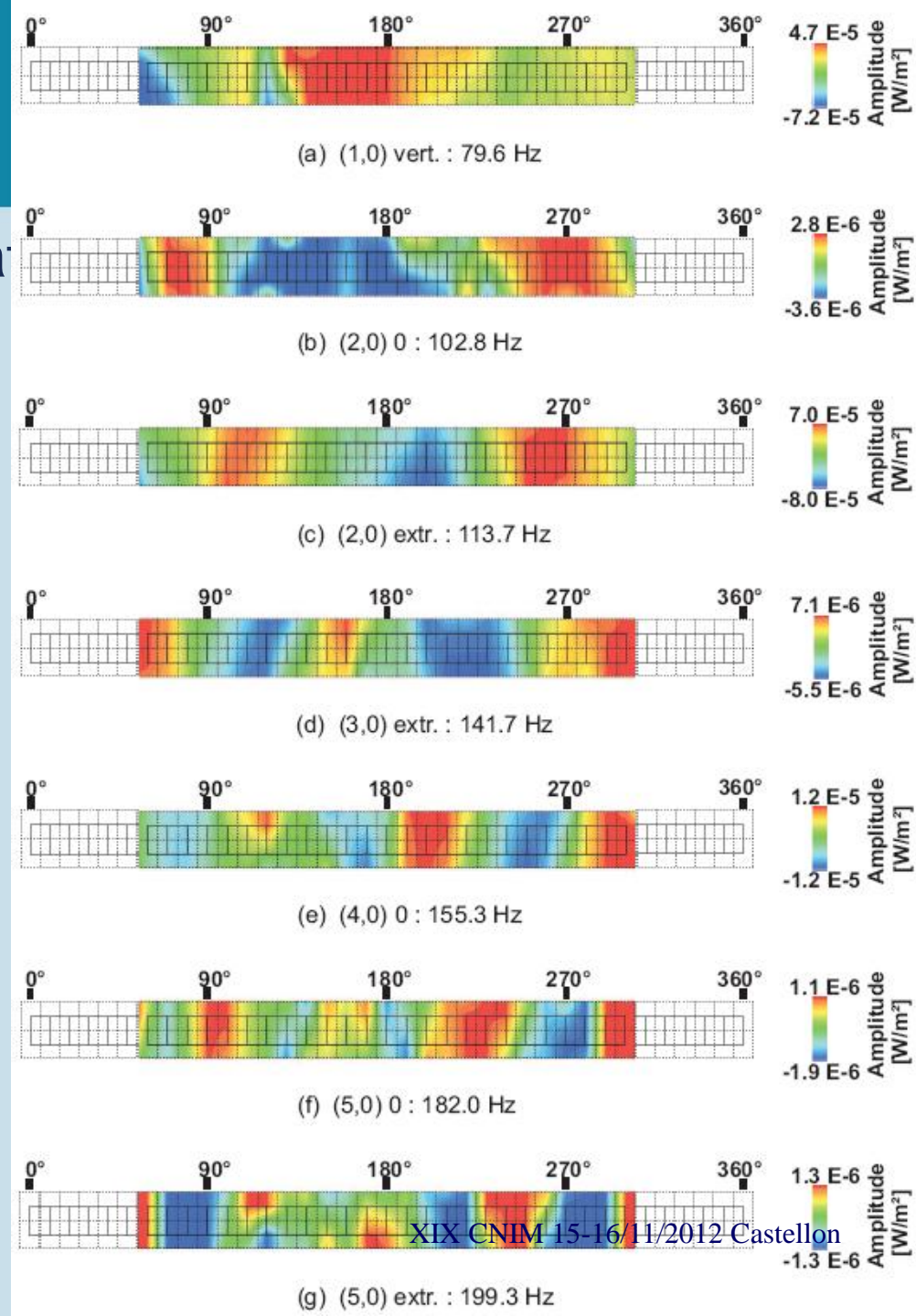
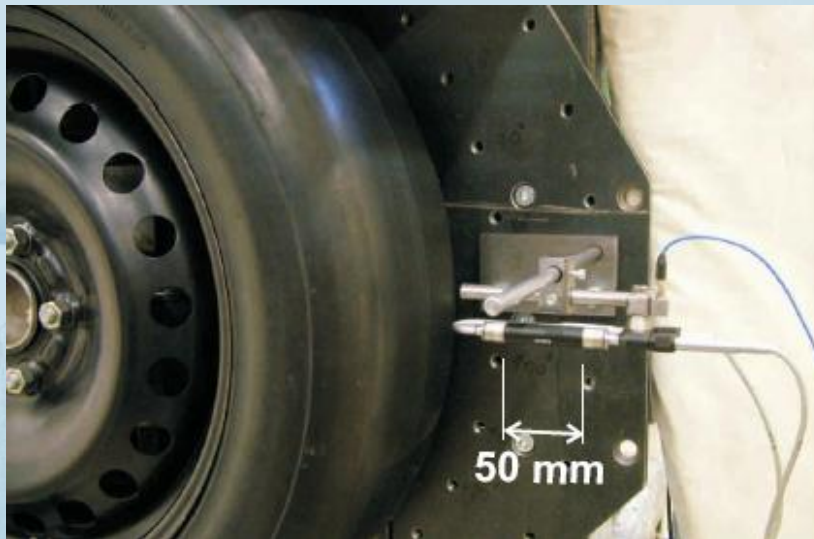


## 3) Change in contact pressure distribution



# Acoustic response

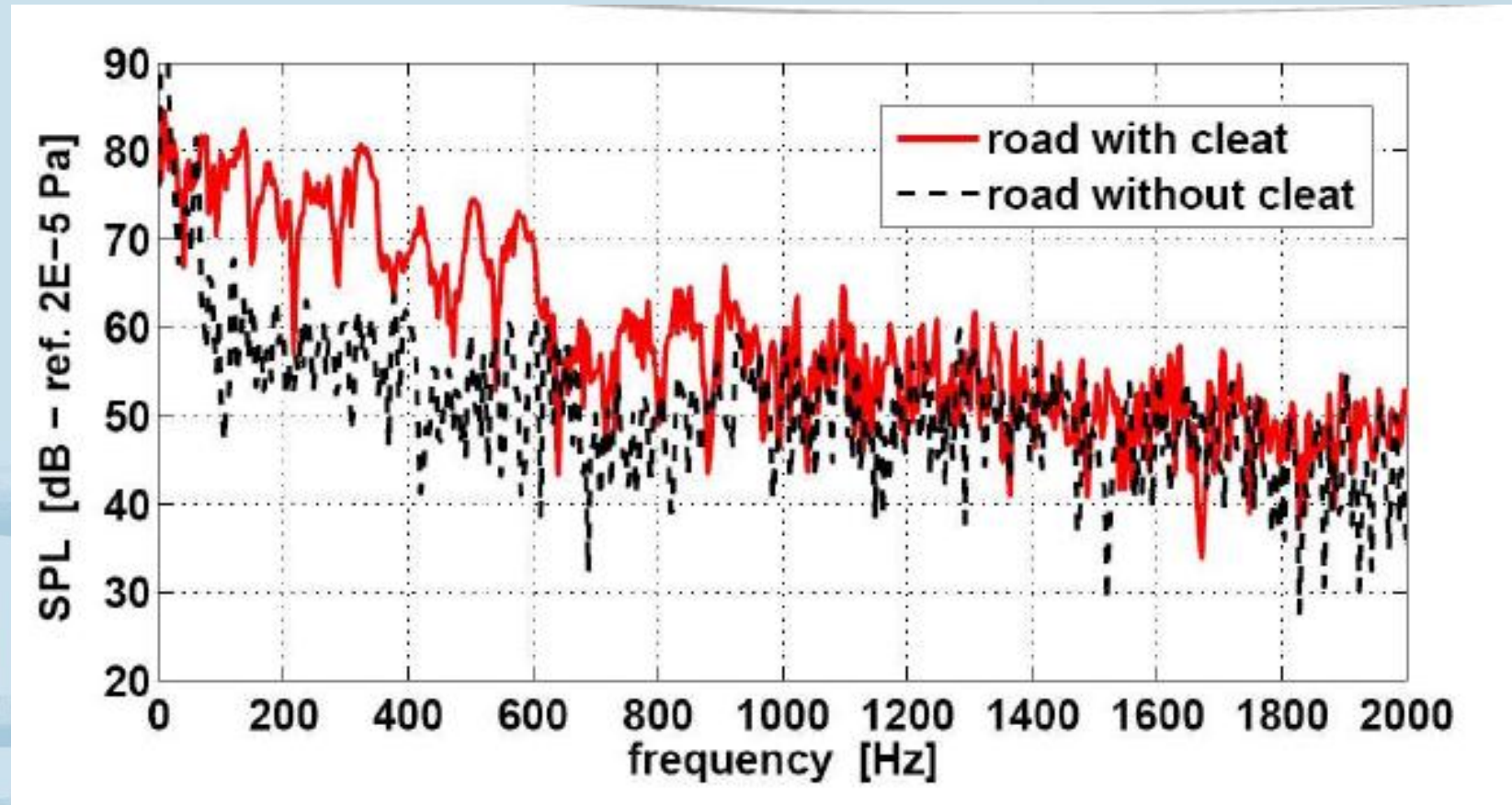
- Sound intensity distribution at the structural resonances
- Treadband causes main structure-borne noise radiation below 300 Hz





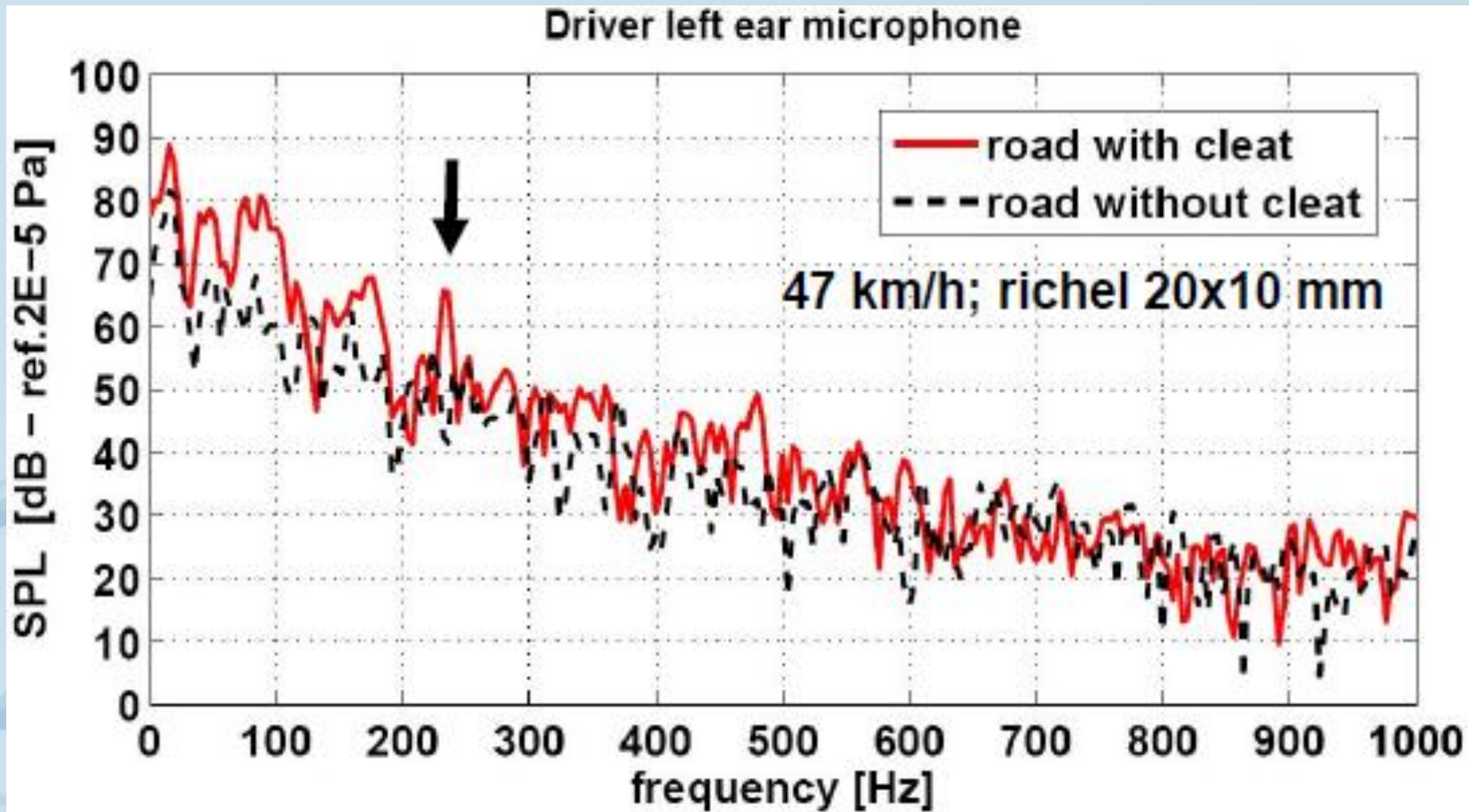
# Acoustic response

- Exterior noise (47km/h), cleat 20x10mm



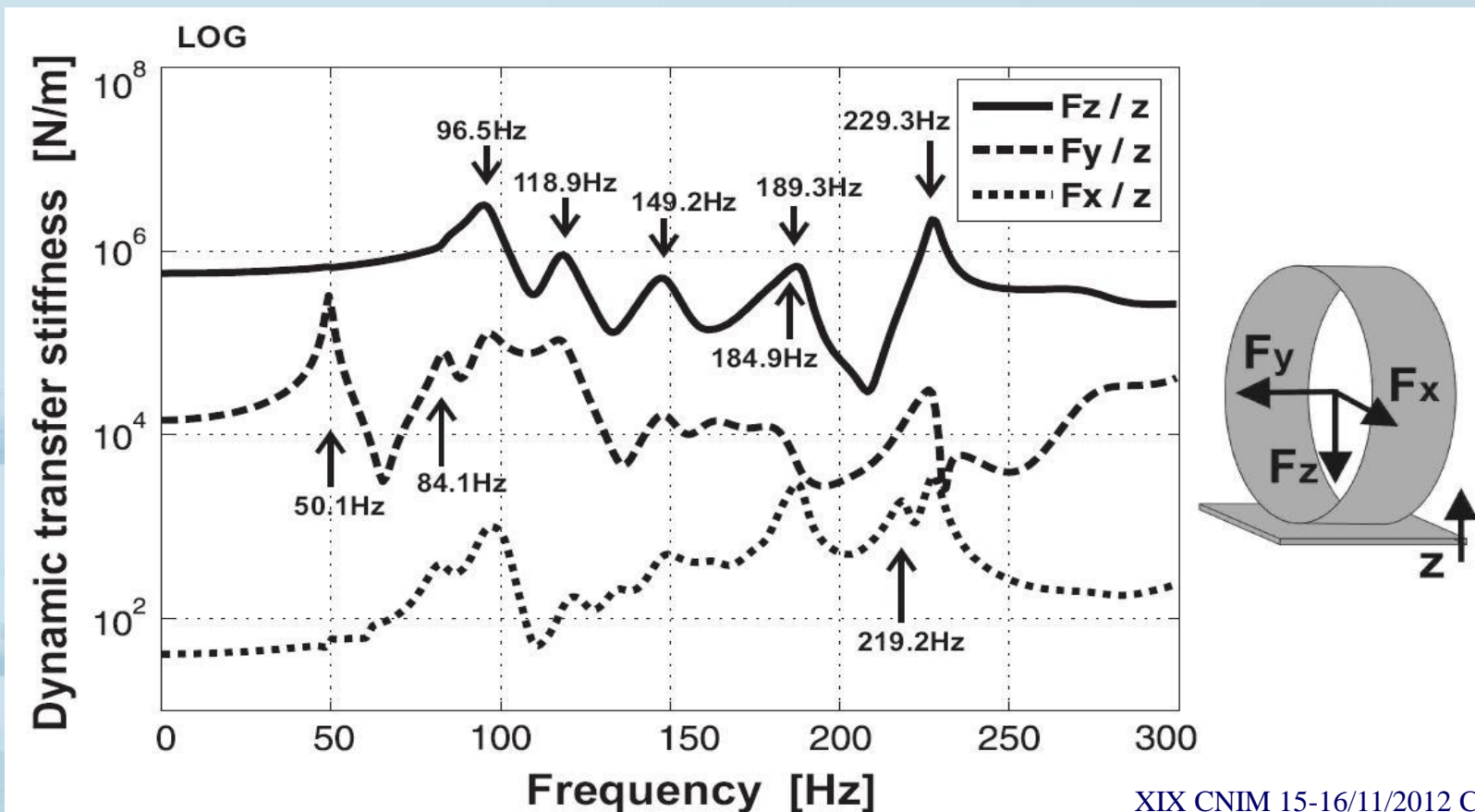
# Acoustic response

- Interior noise (47km/h), cleat 20x10mm



# Tyre dynamic transfer stiffness

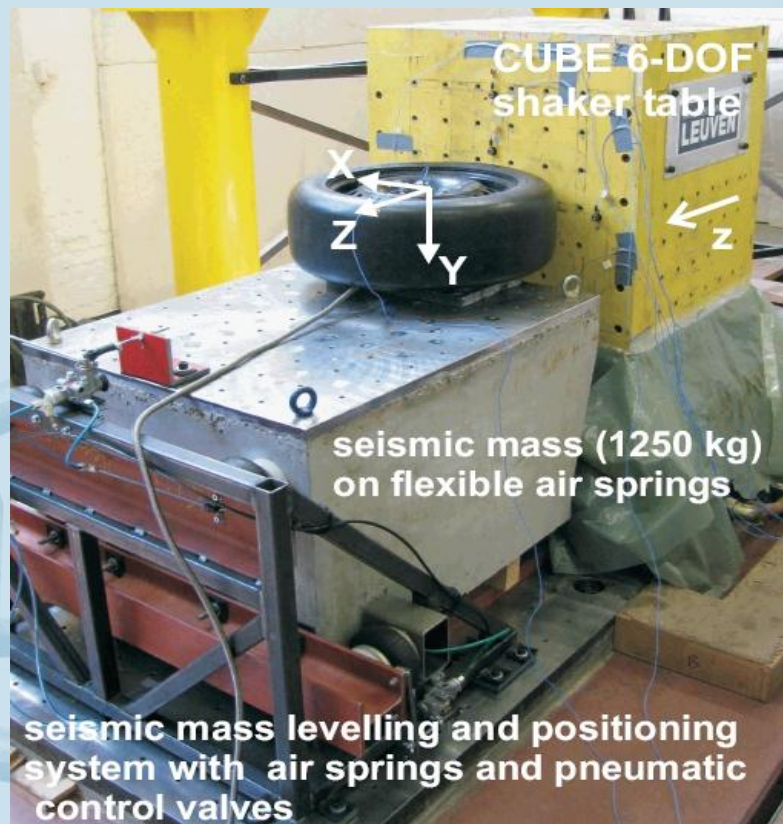
- Important characteristic for structure-borne interior tyre/road noise.



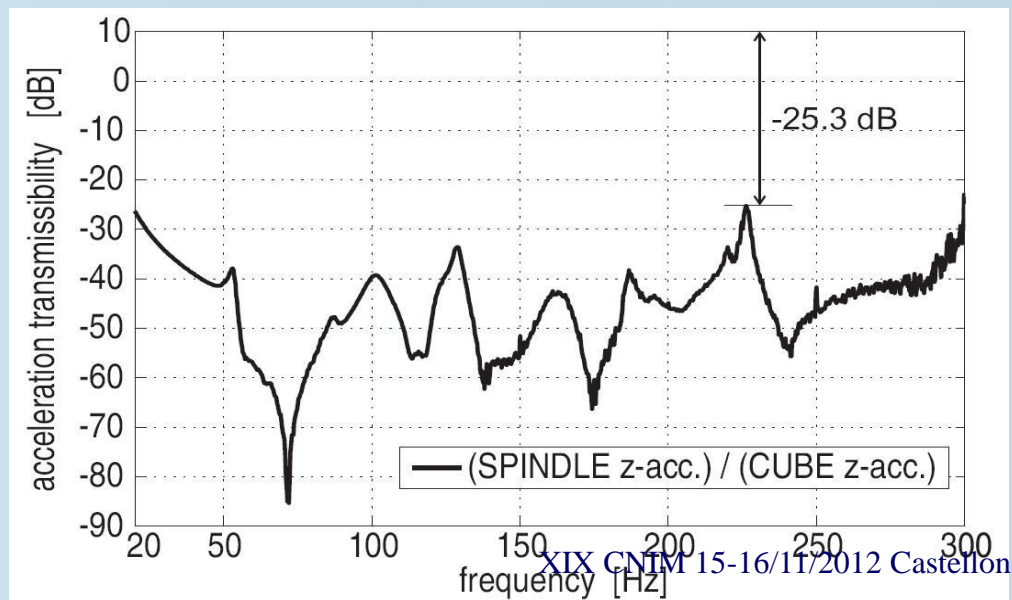


# Tyre dynamic transfer stiffness

- 205/55R16 tire without tread pattern; steel wheel
- dynamic stiffness of a tire that is **rigidly clamped at the spindle**
- ground vibration isolation: seismic mass (1250 kg) supported by four soft air springs (4 x100 kN/m)



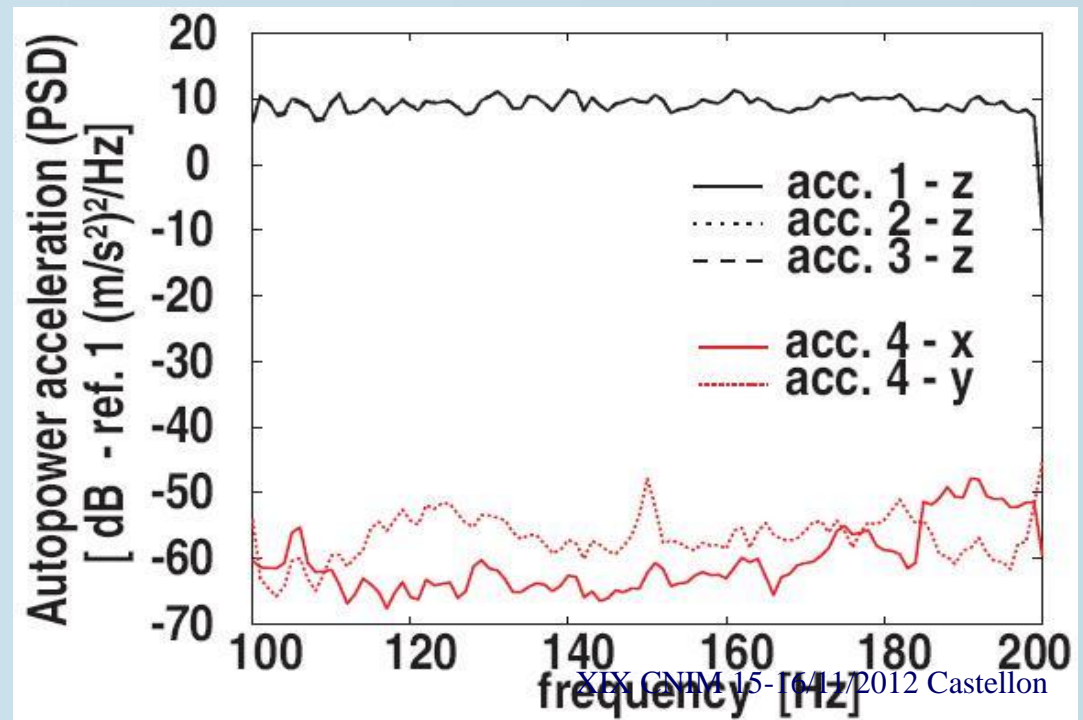
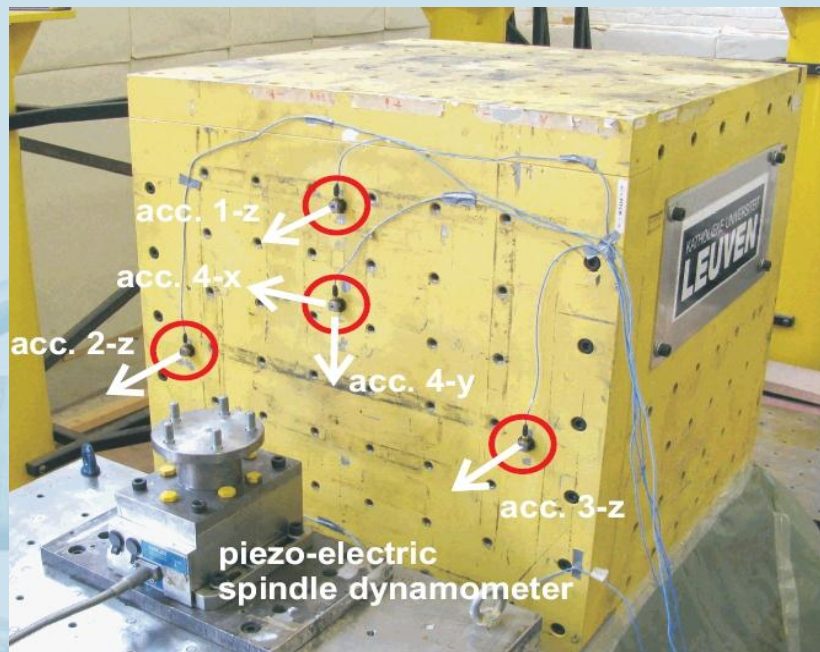
spindle can be considered as rigidly clamped in the frequency range of interest





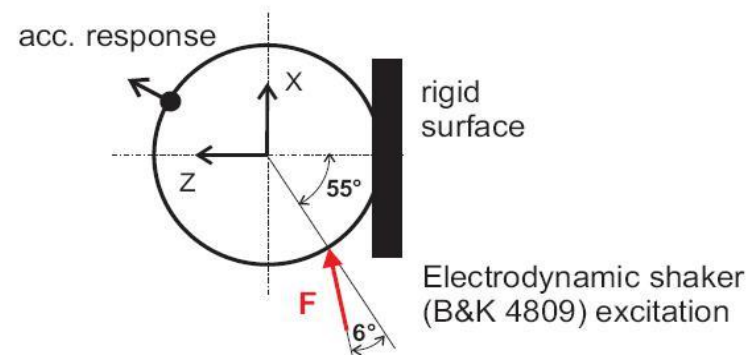
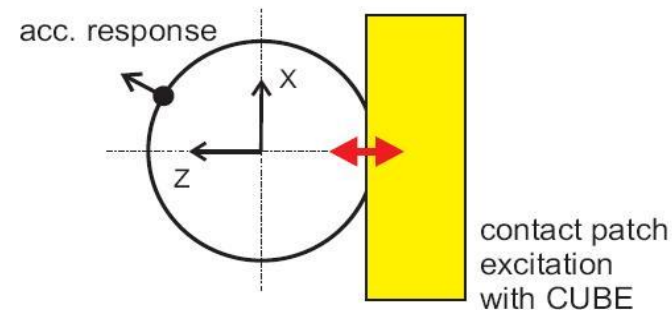
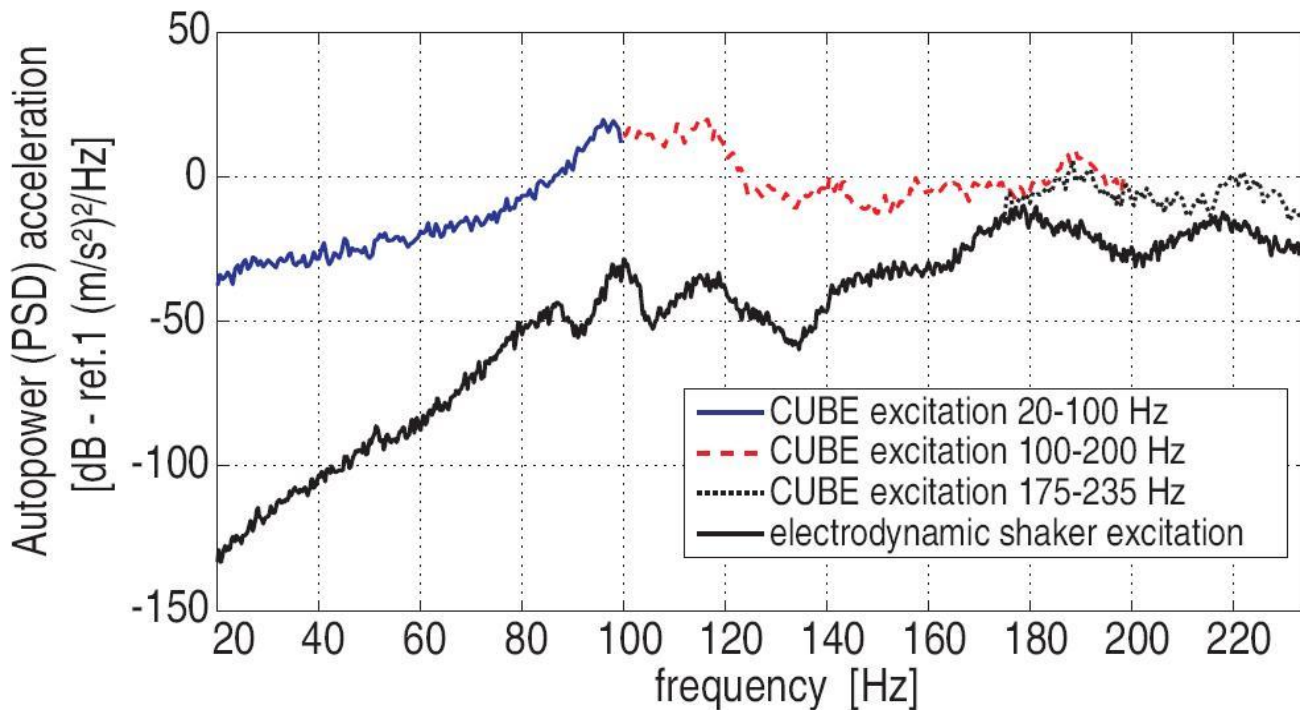
# Test setup layout

- CUBE 6-DOF hydraulic shaker table provides:
  - ➔ static preload (285 kg)
  - ➔ purely **uniaxial** dynamic **random** excitation at the tire contact patch
- motion of the hydraulic shaker table is monitored and controlled through a **Time Waveform Replication (TWR)** algorithm

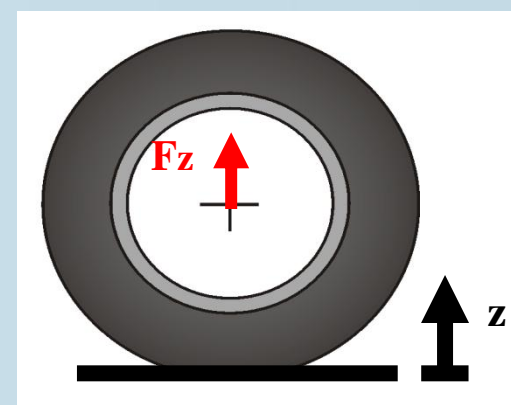
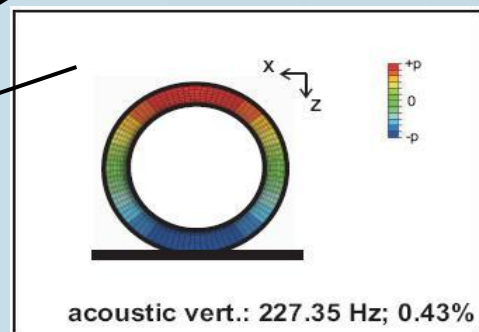
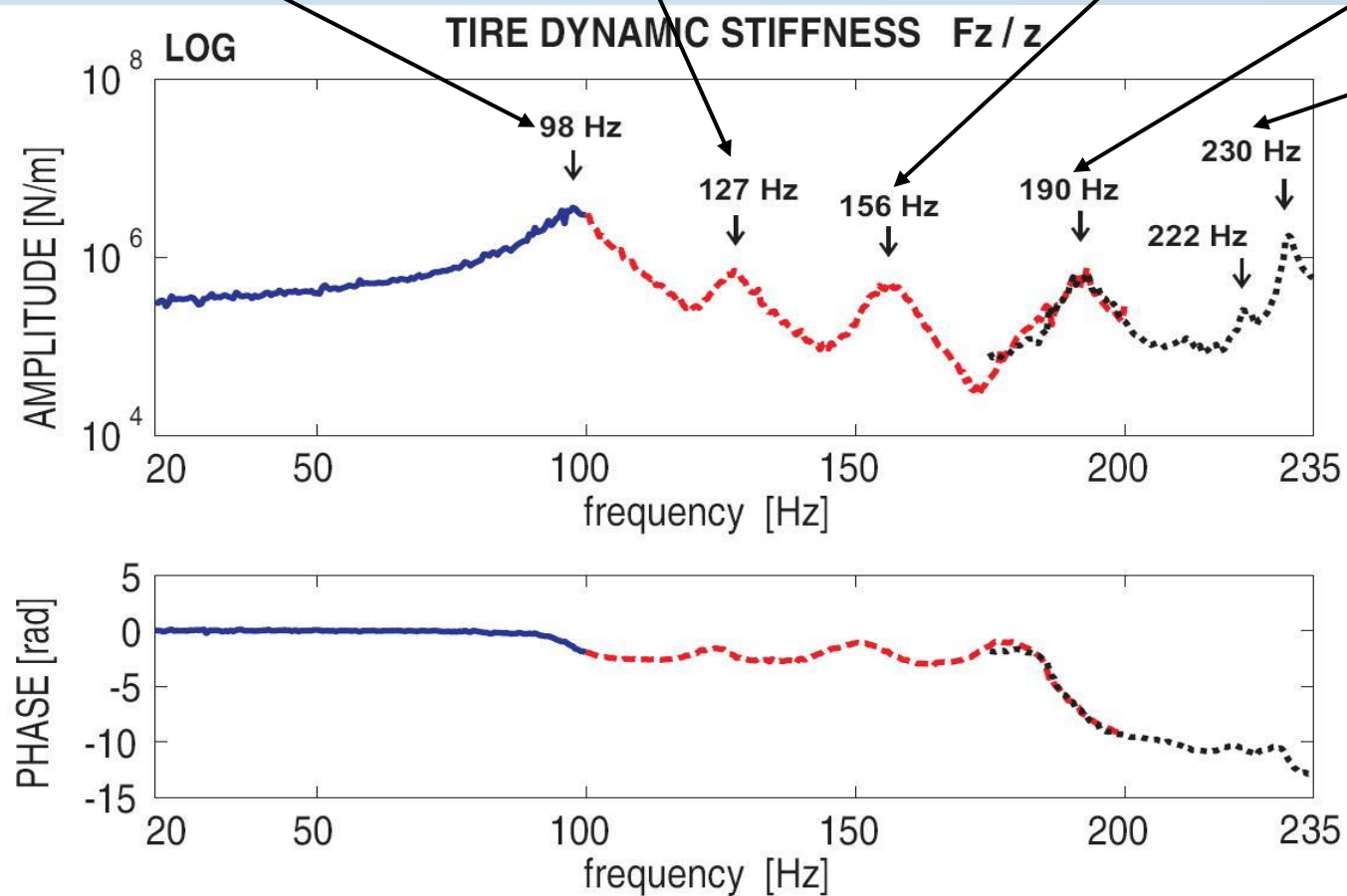
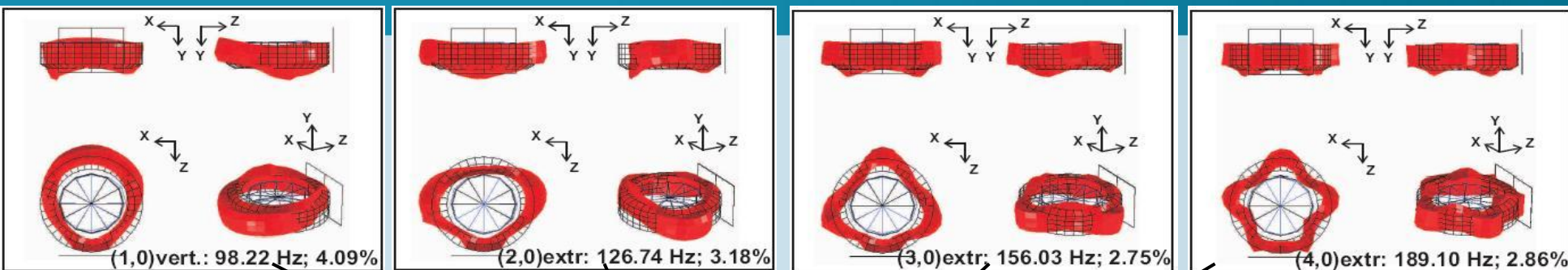


# Operational excitation levels

- measurement of the dynamic transfer stiffness is performed in 3 frequency bands
- higher excitation levels can be obtained in the individual frequency bands

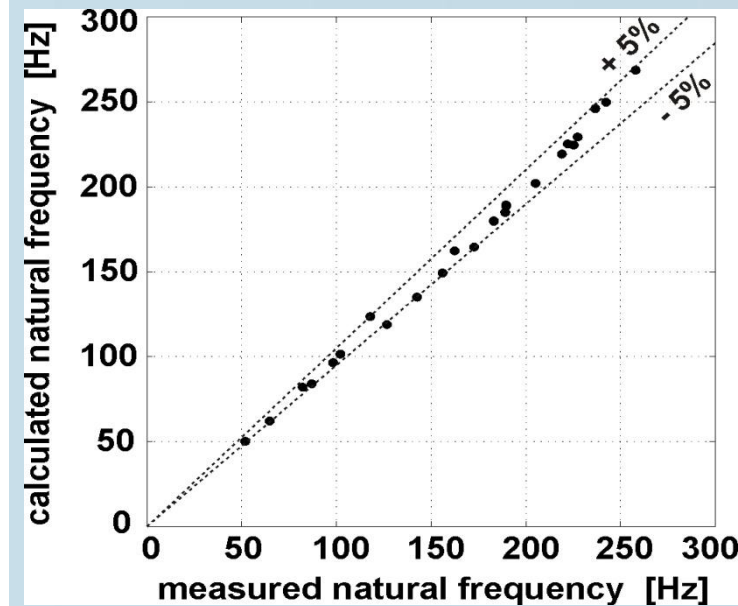
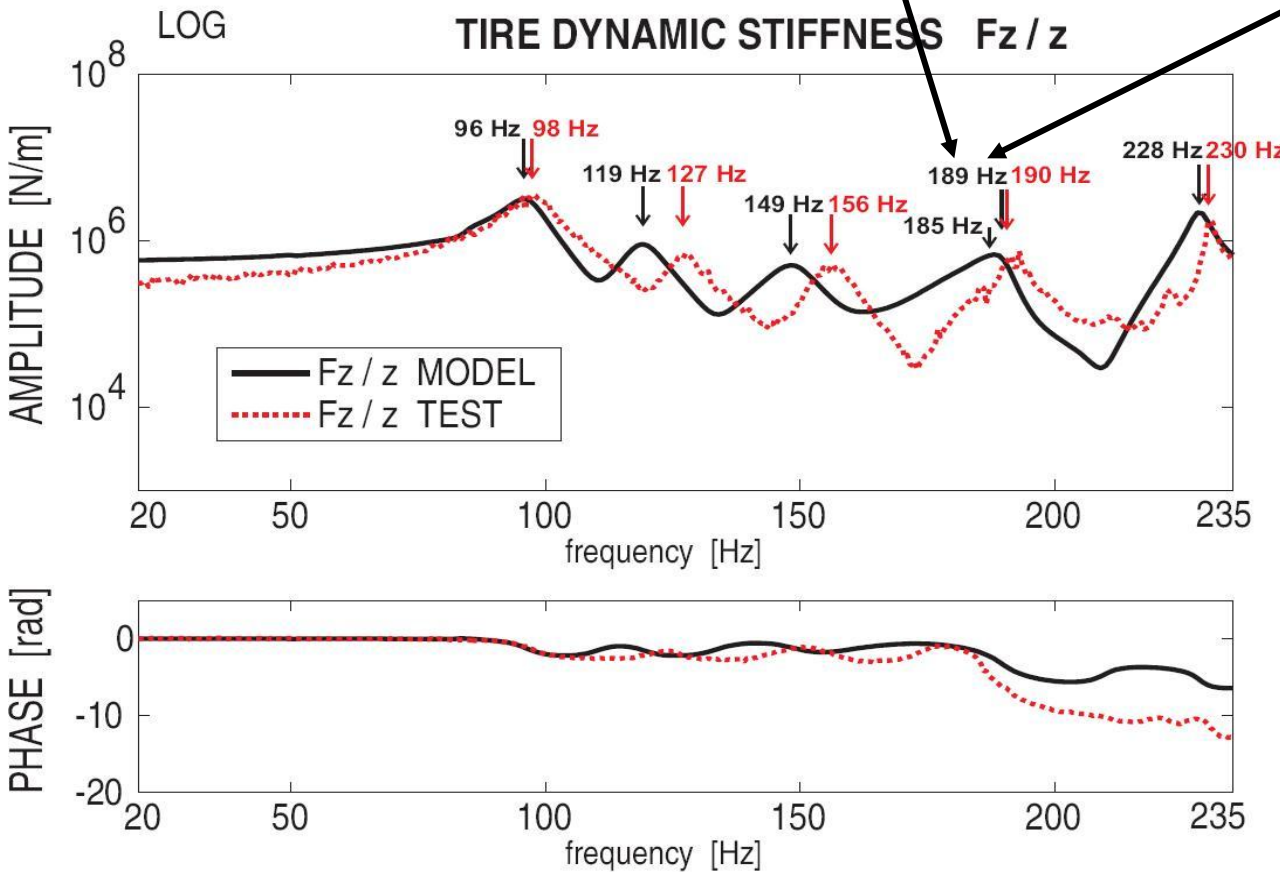
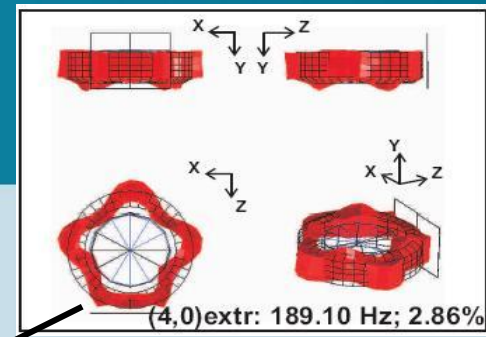
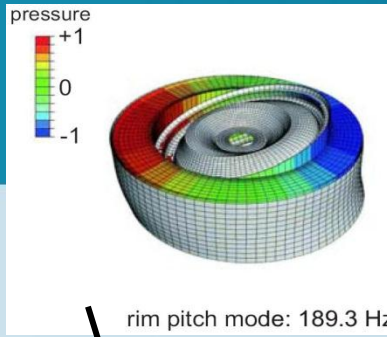


# Measured dynamic transfer stiffness





# Validation

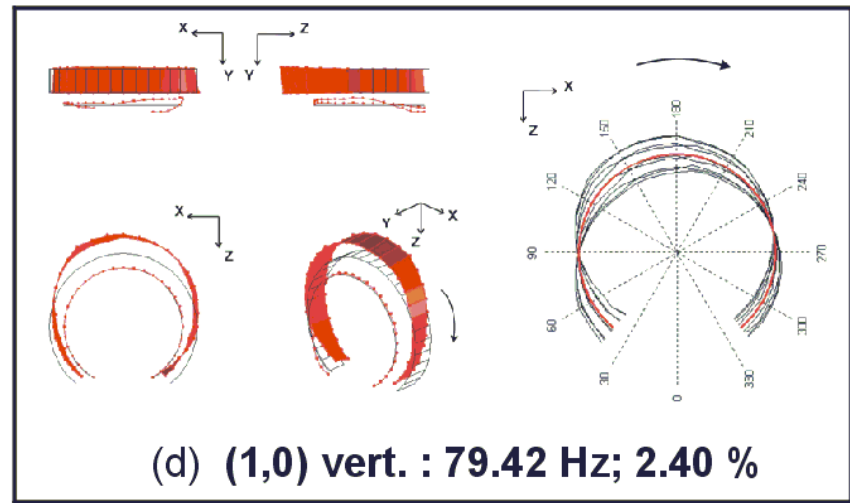
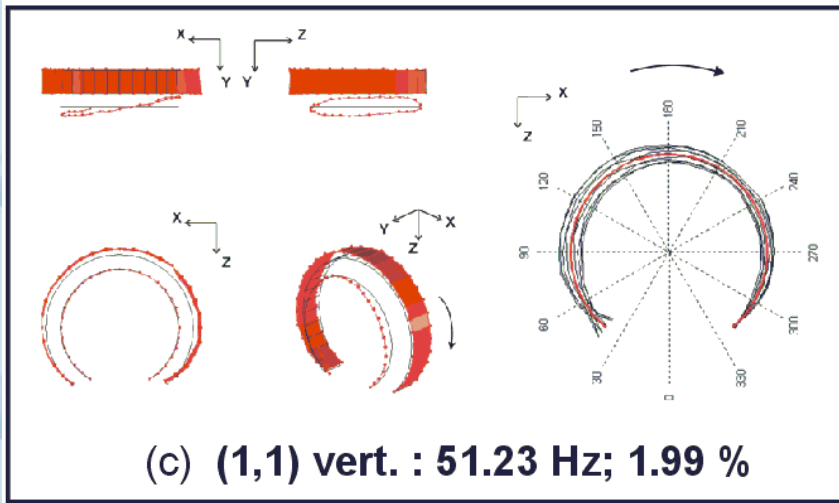
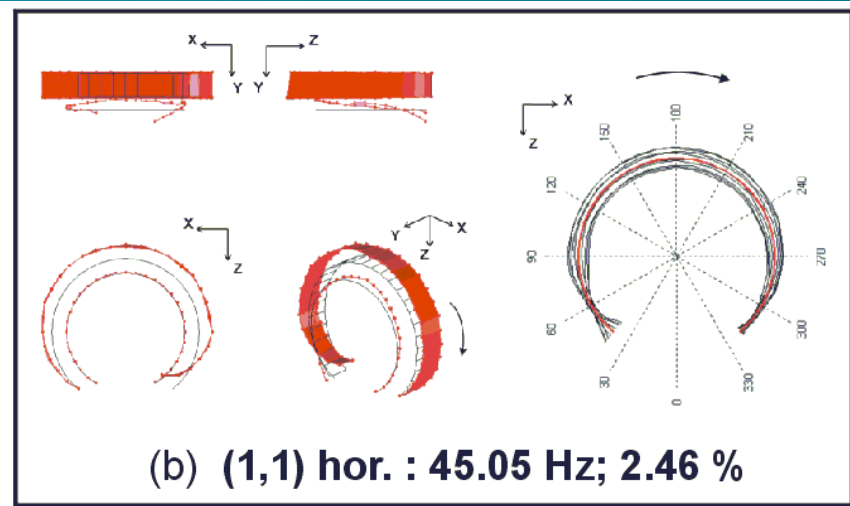
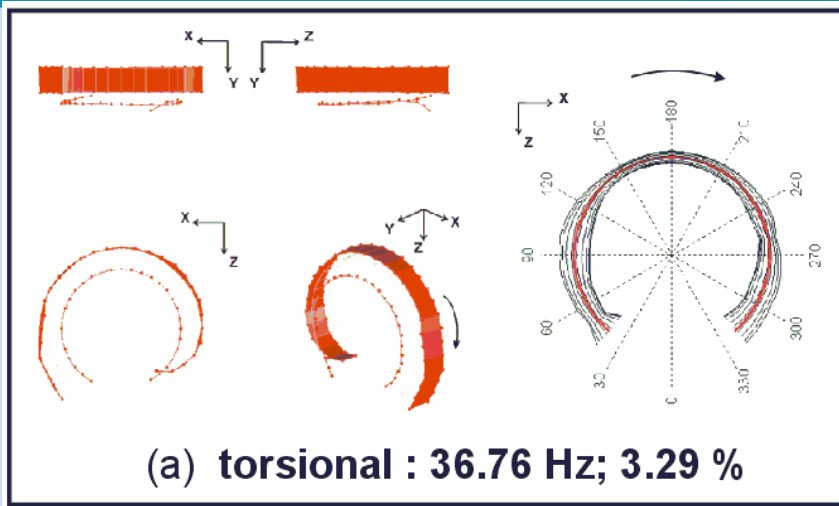




# Questions ?

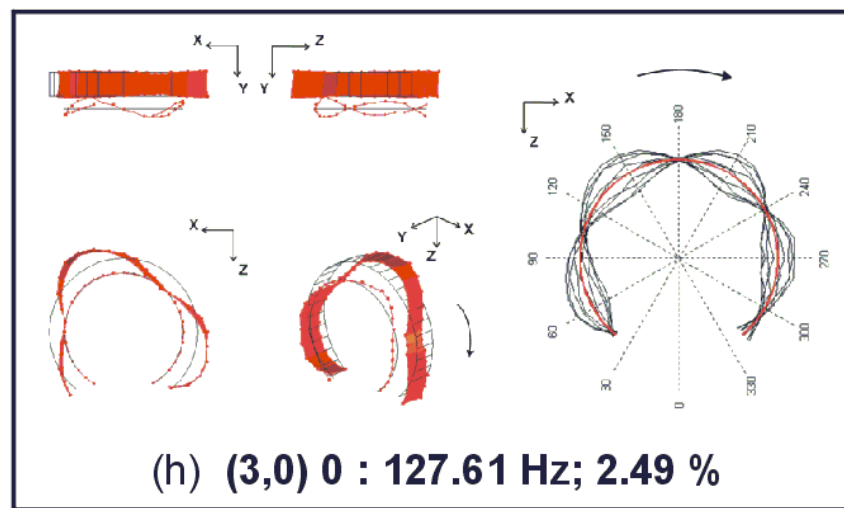
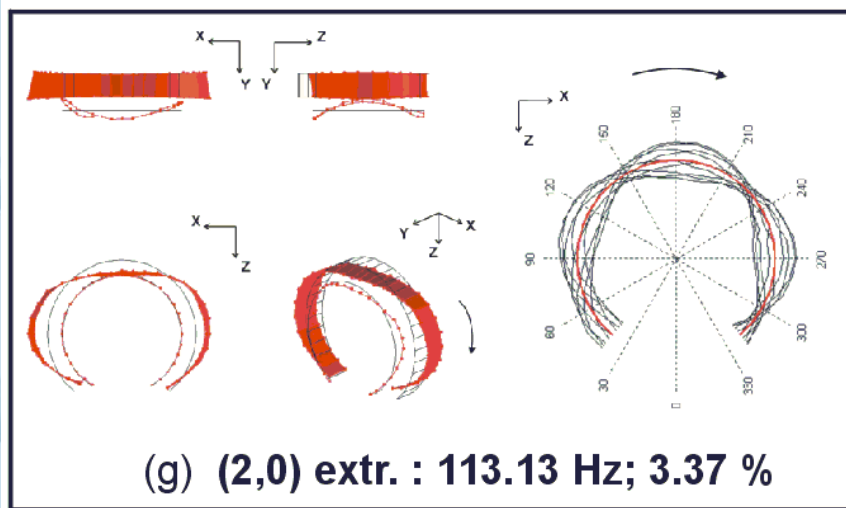
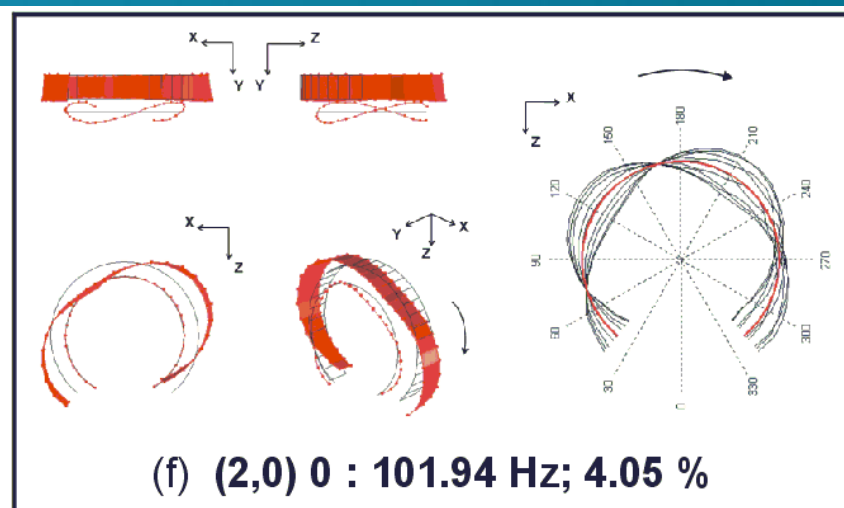
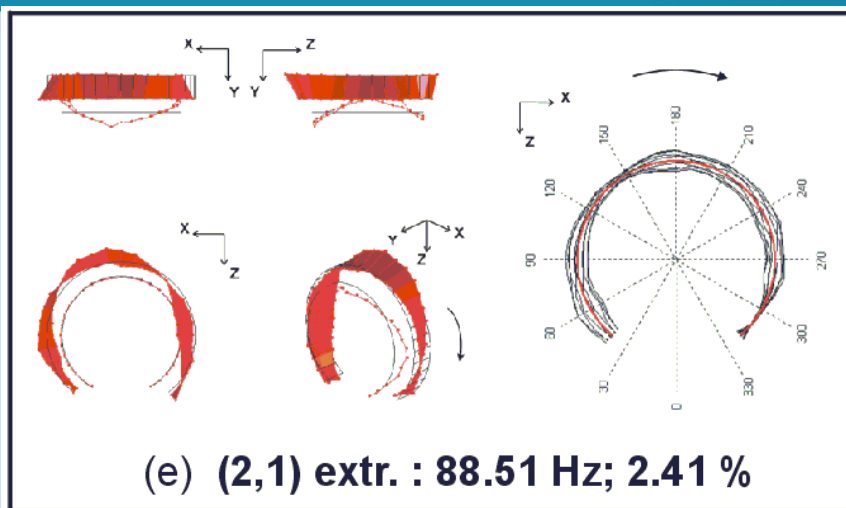


# Rolling tyre Modal Parameters (15.7 rad/s)



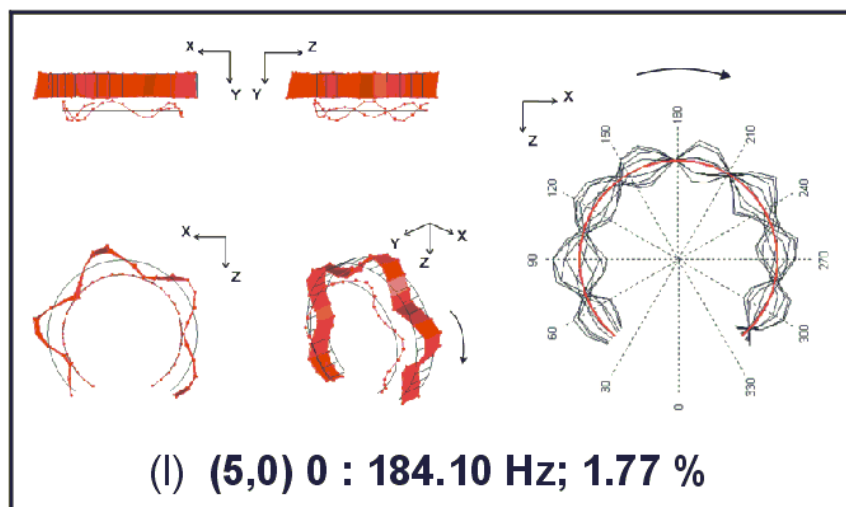
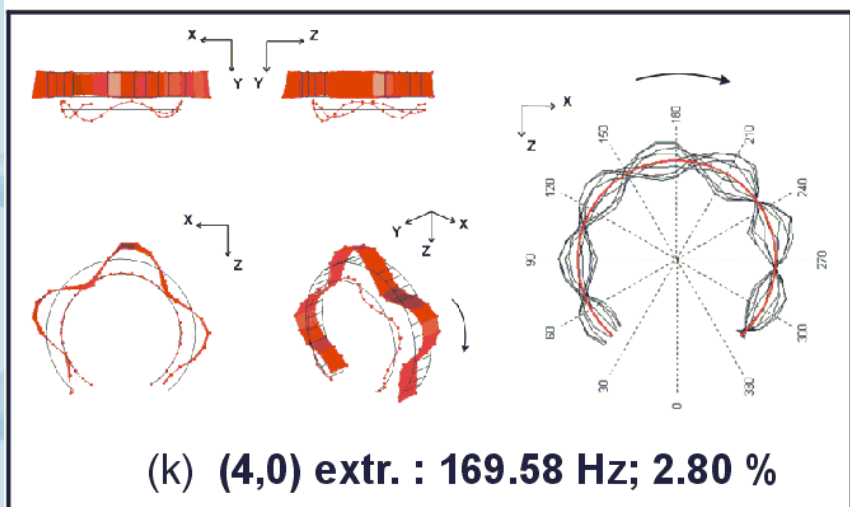
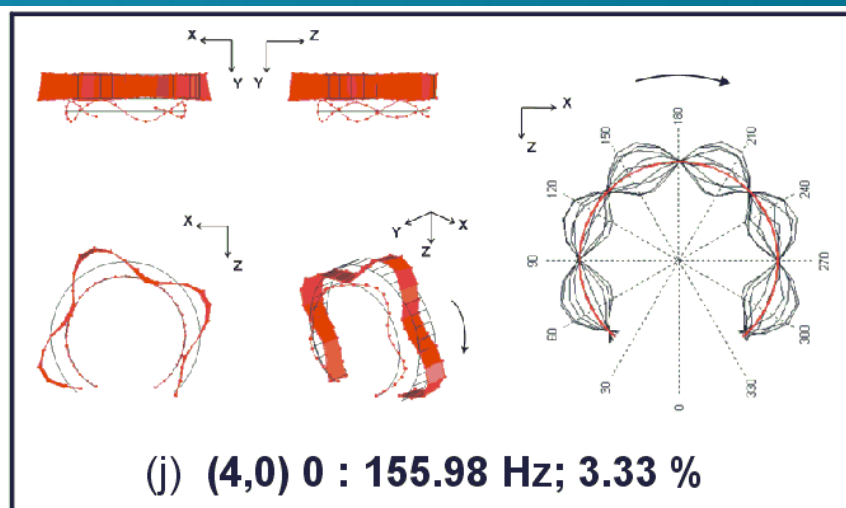
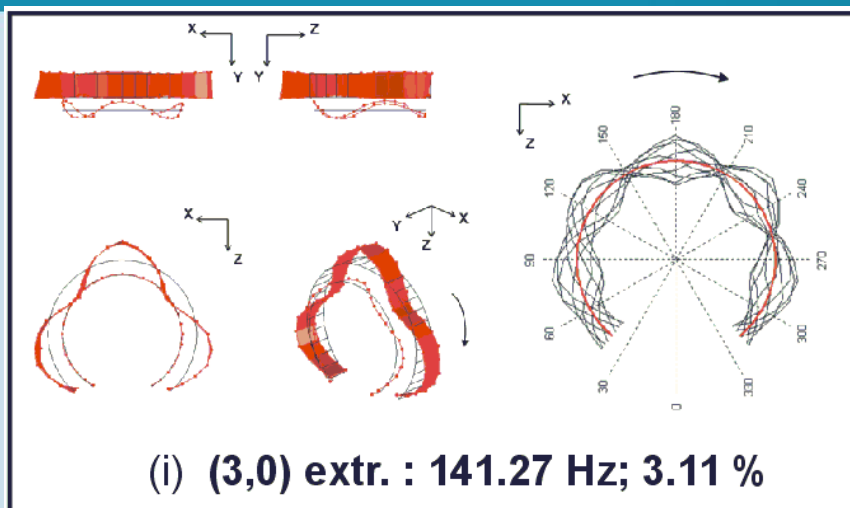
FIXED reference system

# Rolling tyre Modal Parameters (15.7 rad/s)



FIXED reference system

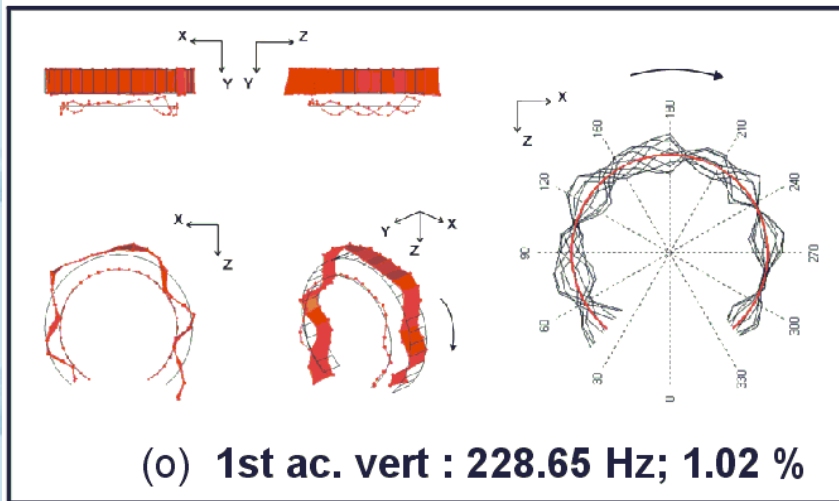
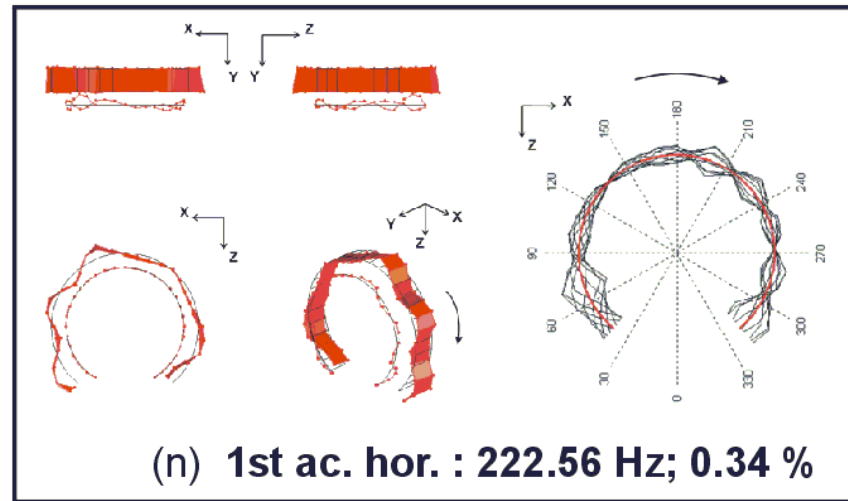
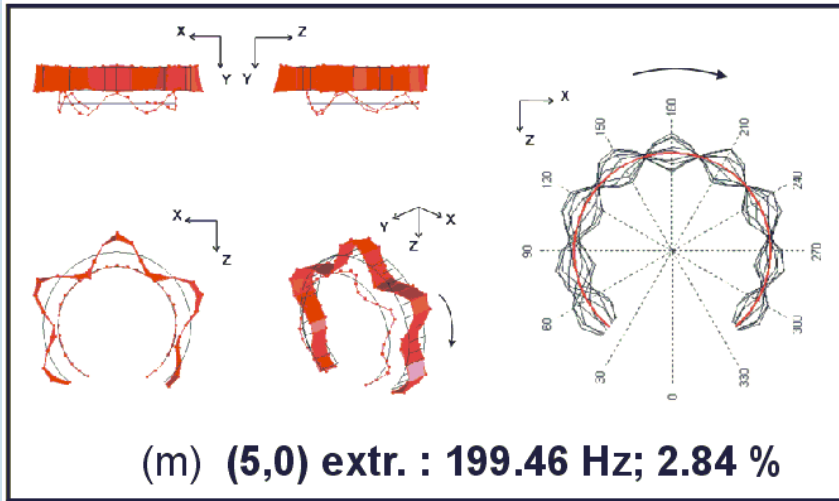
# Rolling tyre Modal Parameters (15.7 rad/s)



FIXED reference system



# Rolling tyre Modal Parameters (15.7 rad/s)



FIXED reference system